Targeting Procedures for Energy Savings by Heat Integration across Plants

Hernán Rodera and Miguel J. Bagajewicz
School of Chemical Engineering and Materials Science, University of Oklahoma, Norman, OK 73019

Heat integration across plants can be accomplished either directly using process streams or indirectly using intermediate fluids. By applying pinch analysis to a system of two plants, it is first shown that the heat transfer leading effectively to energy savings occurs at temperature levels between the pinch points of both plants. In some cases, however, heat transfer in other regions is required to attain maximum savings. A systematic procedure to identify energy-saving targets is discussed, as well as a strategy to determine the minimum number of intermediate-fluid circuits needed to achieve maximum savings. An MILP problem is proposed to determine the optimum location of the intermediate fluid circuits. The use of steam as intermediate fluid and extensions to a system of more than two plants are discussed.

Introduction

Since the onset of heat integration as a tool for process synthesis, energy-saving methods have been developed for the design of energy-efficient individual plants. Heat integration across plants (that is, involving streams from different plants in a complex) always has been considered impractical for various reasons. Among the arguments used is the fact that plants are physically apart from each other and, because of this separation, pumping and piping costs are high. However, an even more powerful argument against integration is the fact that different plants have different startup and shutdown schedules: if integration is done between two plants and one of the plants is put out of service, the other plant may have to resort to an alternative heat-exchanger network to reach its target temperatures. Plants may also operate at different production rates, departing from design conditions and needing additional exchangers to reach desired operating temperatures. All these discouraging aspects of the problem led practitioners and researchers to leave opportunities for heat integration between plants unexplored.

Notwithstanding the aforementioned objections, in several practical instances, these savings opportunities are actually implemented either directly using process streams (Siirola, 1998; Zecchini, 1997), or indirectly through the use of the steam system in what has been called the steam belt (Robertson, 1998). The first attempt to study the recovery of energy through integration between processes was made by Morton and Linnhoff (1984), who considered the overlap of grand composite curves to show the maximum possible heat recovery using steam. Later, Ahmad and Hui (1991) extended this concept to direct and indirect heat integration, also based on the overlapping of grand composite curves. In addition, they proposed a systematic approach to generate different heat-recovery schemes for interprocess integration.

The concept of "total site" was introduced by Dhole and Linnhoff (1992) to describe a set of processes serviced by and linked through a central utility system. Using site-source and site-sink profiles (based on the combination of modified grand composite curves of the individual processes), they set targets for the generation and use of steam between processes. However, the elimination of process-to-process heat-exchange zones, also called "pockets," from the grand composite curves of the individual processes reduces in certain cases the opportunities for energy recovery. This point is discussed in detail in the present article.

One of the questions in total-site integration is whether a process fluid should be used to perform the heat transfer or an intermediate fluid should be used. In addition, the question of how to preserve energy efficiency when nonsimultaneous shutdowns take place needs to be addressed. The objective then is to have a dual design where both heat integration and

Correspondence concerning this article should be addressed to M. J. Bagajewicz.

independent operation are achievable. In this first approach to the problem, energy targets are established. The nonsimultaneous operation of each of the plants is directly related to the ability of the heat-exchanger network in both plants to operate in either of the other modes. This multiobjective design procedure will be presented in detail in a follow-up article.

In principle, direct transfer of heat from one plant to the other may involve many process streams, which results in many heat exchangers. The incentives to use intermediate fluids are the following.

- *Multiple pumps and compressors.* Integration across plants may require the transferring of heat from a number of streams in one plant to a number of streams in the other. Thus, the cost of integration can be high because of the use of multiple pump and compressor units.
- *Pumping and compression costs.* A fluid with a larger heat capacity than the process streams will result in a smaller flow rate of liquids to pump among plants. When process streams are gases, then the installation of compressors to cover large distances can be more expensive than the equivalent pumping of an intermediate fluid.
- *Safety.* Process streams being pumped large distances may pose a hazard, should any spill occur.
- *Control.* Piping process streams long distances also introduces long delays, which would eventually make process control more difficult. The use of intermediate fluids simplifies the problem.

There are, however, some disadvantages worth mentioning.

- The use of an intermediate fluid reduces the interval of effective heat transfer (that is, between pinches) by a multiple of the minimum temperature difference ($\Delta T_{\min}$). Therefore, compared with the direct integration case, smaller savings can be obtained.
- The number of heat exchangers involved in a setup that uses intermediate fluids can also be higher than using direct heat exchange. In this case, in the absence of other incentives, the trade-off is between the new number of heat exchangers and the pumping costs.

In many cases, steam can be used as an intermediate fluid. This offers many possibilities, as the steam system is already in place. Recent work regarding the use of the utility system for the indirect integration of different processes (Hui and Ahmad, 1994) focuses on the generation and use of steam to reduce utility costs. The present work introduces targets based on fixed steam pressures that are usually available in the plants.

In this article, pinch analysis is used to establish target maximum energy savings for either direct or indirect integration of the case of two plants. The article is organized as follows: temperature intervals where heat transfer should take place are identified first, together with the identification of which plant should be the source. An LP problem is set up to determine these targets. The design of the intermediate fluid circuits is considered next. The possibility of using a single intermediate fluid circuit is evaluated. An MILP model is introduced to determine the location of the minimum number of circuits needed to achieve the target savings. Extensions to the problem of integration of a set of n plants are briefly discussed. To illustrate these concepts, examples using heat-integration problems from the literature are solved. In addition, an example consisting of the integration between a crude unit and an FCC plant is solved.

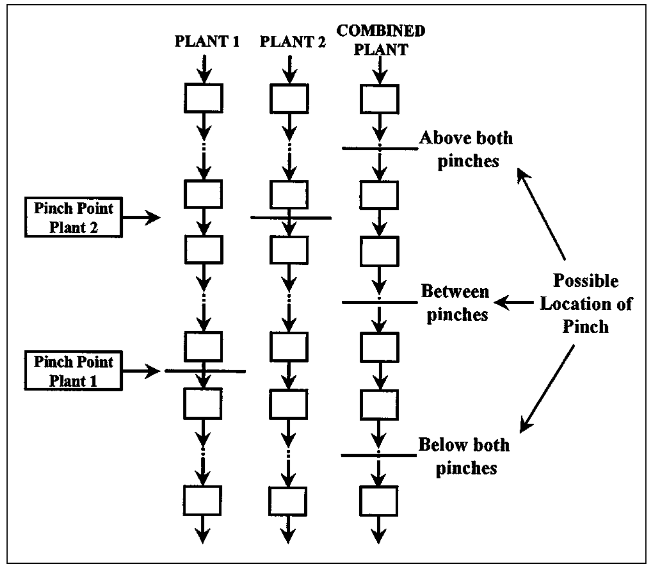

Figure 1. Possible location of combined plant pinch point.

Maximum Transferable Heat

Consider two plants and suppose that minimum utility targets are obtained independently for each of the plants using LP transportation or transshipment models (Cerda et al., 1983; Papoulias and Grossmann, 1983). When all the streams from both plants are included in the same set (combined plant), the total minimum heating and cooling utility targets are usually lower.

Location of the combined-plant pinch

Without loss of generality, assume that plant 2 has a pinch temperature that is higher than the pinch temperature for plant 1. Therefore, the pinch of the combined plant can fall in any of these three regions: (a) above the pinch of plant 2; (b) between the pinches; or (c) below the pinch of plant 1 (Figure 1).

Consider first the case in which all intervals above the pinch of plant 2 in each of the individual plants are sinks of heat. If the corresponding intervals of both plants are added, the combined plant will also have sinks in these intervals. Therefore, the system considered has a combined pinch located no higher than the original pinch of plant 2. A similar analysis can be made for the region below the pinch of plant 1. When the preceding conditions are not met, the combined pinch can be located in any place. The implications of this will be investigated further.

The first intuitive conclusion one can make is that heat should be transferred at temperatures between the pinch points of the original plants. Indeed, this is the only region where plant 2 is a heat source while plant 1 is a heat sink. This intuitive conclusion is in principle correct, but sometimes heat transfer in the opposite direction (from plant 1 to plant 2) above or below both individual plant pinches is required to assist the realization of maximum savings.

Transfer of heat outside the region between both pinch points

Transferring heat either above the higher pinch or below the lower pinch does not decrease the utility usage. Only an equivalent amount of corresponding utility is shifted from one plant to the other. Figure 2 illustrates the effect of an amount of heat Q_A transferred from plant 2 to plant 1 in the upper zone (without loss of generality, intervals are lumped to allow clarity of illustration). An increase of the heating utility in the plant that releases the heat is followed by a reduction of the same utility in the plant that receives the heat. The same effect on the cooling needs is observed if a certain amount of heat Q_B is transferred from plant 2 to plant 1 in the lower zone (Figure 2). In addition, as the temperature level at which the heat transfer in the upper zone takes place is lowered, a maximum that can be transferred exists. For example, if the transfer is made in the first interval, the amount that can be transferred is constrained by the original utility usage of plant 1, $S^{\mathrm{I}}_{\min}$. If the heat is transferred in an interval below the first one, say interval i, the upper limit will be smaller, as some of the utility used by plant 1 is used to satisfy the heat demand of the first $i-1$ intervals. Therefore, to compute this upper limit one should subtract all the intervals that are heat sinks (negative values) above the interval of transfer from $S^{\mathrm{I}}_{\min}$. Similar upper limits for the transfer of heat are found if the lower zone is considered.

In conclusion, no savings can be obtained by transferring heat in the regions above the higher pinch or below the lower pinch. However, transfer from plant 1 to plant 2 in either one of these regions is needed in some cases to facilitate the transfer of heat in the region between both pinch temperatures. This is explored next.

Transfer of heat between both pinch points

As illustrated in Figure 3, a certain amount of heat Q_E is transferred from plant 2 to plant 1 between pinch points. This

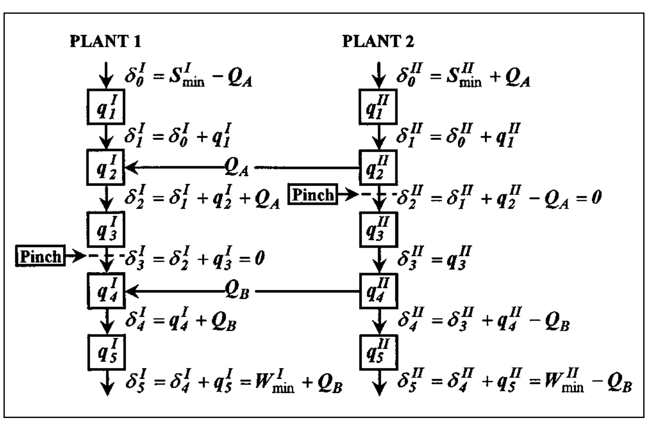

Figure 2. Effect of transferring heat outside the region between both pinch temperatures.

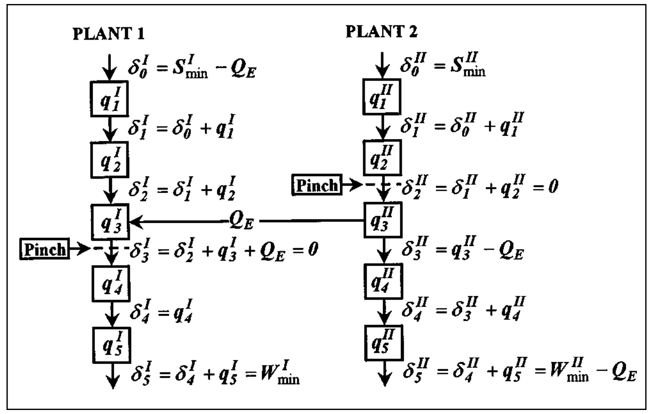

Figure 3. Effect of transferring heat in the region between both pinch temperatures.

transfer has the effect of reducing the heating utility in plant 1 and cooling utility in plant 2. In addition, transferring heat from plant 2 to plant 1 has the effect of reducing the lowest level of the heating utility demand on plant 1, which is usually the cheapest. Finally, it has no effect on the heating utility of plant 2 or the cooling utility of plant 1.

Assisted and unassisted heat transfer across plants

We now investigate the upper limits in the amount of heat that plant 1 can accept and in the amount that plant 2 can deliver in the region between pinches. Consider the case in which there are only sink intervals above the pinch point of plant 2 and only source intervals below the pinch point of plant 1. The maximum heat that plant 1 can receive is the actual sum of the demands it has in the intervals between pinches. Similarly, the maximum amount that plant 2 can transfer is the resulting available heat it has between pinches. Since any heat that is transferred to plant 1 at any temperature interval can be cascaded down to lower temperatures, the real limitation on how much can be transferred is given by the ability of plant 2 to fulfill the demand at each interval. Because all the intervals above the pinch point of plant 2 are sinks, the whole demand of the heat in plant 1 is only satisfied by utility or by plant 2 from the intervals between pinches. Likewise, since all intervals below the pinch point of plant 1 are sources of heat, plant 2 does not need to use heat from the intervals between pinches to satisfy any demand below the pinch point of plant 1. Therefore, the amount of heat that can be transferred to plant 1 is not limited by such demand. This motivates the following definition:

Definition. Unassisted heat transfer across plants takes place when only heat transfer between pinches is needed to achieve maximum savings.

A special case of unassisted heat transfer across plants is when plant 1 has only sink intervals above the pinch point of plant 2 and only source intervals below the pinch point of plant 1. Unassisted cases can also take place even though some intervals in plant 1 are sources of heat above the pinch of plant 2 or some intervals in plant 2 are sinks of heat below the pinch of plant 1. If a case is unassisted, the combined

Table 1. Example 1

Temp. Scale	Plant 1			Plant 2			Combined Plant		
140	q_i^{I}	γ_i^{I}	S_{min}^{I}	q_i^{II}	γ_i^{II}	S_{min}^{II}	q_i^{CP}	γ_i^{CP}	S_{min}^{CP}
120	−12	−12	**30**	−19	−19	**20**	−31	−31	**38**
100	−1	−13		−1	−20		−2	−33	
80	−15	−28	W_{min}^{I}	10	−10	W_{min}^{II}	−5	−38	W_{min}^{CP}
60	−2	−30		5	−5		3	−35	
40	2	−28	**2**	1	−4	**16**	3	−32	**6**

Max. potential savings between pinches = **12** Max. possible savings = **12**

pinch point of both plants lies in between pinches. Indeed, the addition of all intervals and the fact that in general there is heat transfer across the location of the pinch of plant 2 indicates that the pinch does not lie above this temperature. The same can be said for the region below the pinch point of plant 1.

Assume now that some of the intervals in plant 1 above the location of plant 2 pinch are sources of heat. Furthermore, assume such heat sources are enough to produce a surplus that in the absence of integration across plants is effectively transferred in plant 1 through the location of the pinch point of plant 2. In other words, the surplus of heat above the pinch of plant 2 needs to be used to satisfy the heat demand of plant 1 between pinches. In turn, this may limit the amount that can be transferred from plant 2, and therefore limit the maximum savings that can be obtained. To prevent such limitation, one can transfer the surplus heat from plant 1 to plant 2, reducing the heating utility of plant 2, and allowing maximum heat transfer between pinches. The heat transfer outside the region between pinches does not realize any savings, only shifts utility load from one plant to the other. In fact, if the surplus is larger than the heating utility of plant 2, the amount Q_A is limited by S_{min}^{II}, and the surplus may become an effective limitation to realize all the potential for savings.

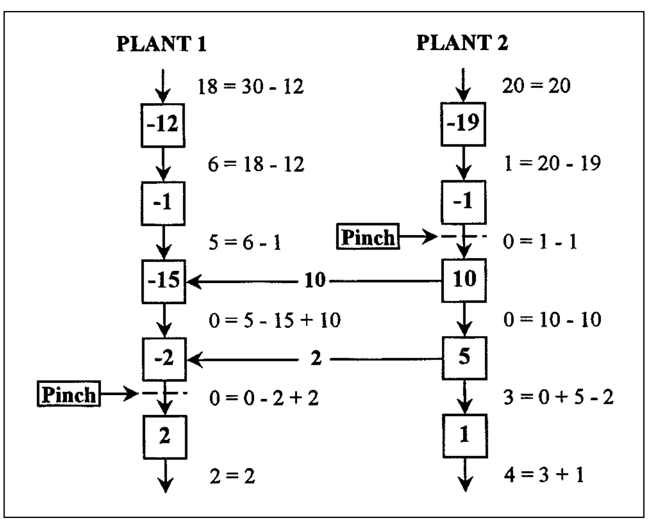

Figure 4. Cascade diagram solution for Example 1.

Similarly, if the heat demand of plant 2 in the corresponding intervals is not sufficiently large, the total surplus that can be transferred is limited. An exact symmetric case happens below the pinch of plant 1. Some of the surplus from plant 1 below its pinch can eventually be used to satisfy this demand, thus freeing the heat from plant 2 to be completely available to realize savings through transfer to plant 1 between pinches.

These two cases motivate the following definition.

Definition. Assisted heat transfer across plants takes place when heat transfer between pinches needs to be assisted by heat transfer outside this region to attain maximum savings.

The existence of assisted cases has been overlooked by Ahmad and Hui (1991), who only showed that sometimes more than one steam level is required for maximum indirect recovery between processes. However, they do not explore further the significance of the assisted transfer in order to realize maximum savings. Dhole and Linnhoff (1992) construct site source and site sink profiles based on the combination of modified grand composite curves of the individual processes. In these modified curves process-to-process heat-integration zones or pockets are eliminated. Consequently, in the presence of an assisted case, opportunities for realizing maximum savings are lost and only limited savings between pinches can be pursued.

A model to predict the exact amount of heat that needs to be transferred in each region is presented later. First, some illustrative examples are shown.

Example of unassisted case

Table 1 shows the interval balances, the heat cascade to determine the utilities, and the actual value of these utilities for each of the plants as well as for the combined plant. Sink intervals are located above the pinch and source intervals are located below the pinch in either plant 1 or plant 2. Therefore, this is an unassisted case, and only transfer between pinches is needed in order to obtain maximum savings. These savings are obtained by subtracting from the sum of the individual utilities the combined utility. Pinch locations are shown with filled lines. As expected, the combined pinch is in between the original plant pinches. Figure 4 shows the cascade diagram solution after the integration is conducted.

In order to compare the results obtained using the cascade diagram with methods that make use of grand composite

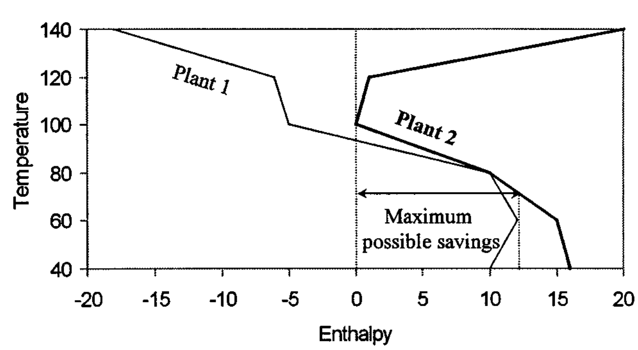

Figure 5. Countercurrent composite curve profiles for Example 1.

Table 2. Example 2

Temp. Scale 140	Plant 1			Plant 2			Combined Plant		
	q_i^{I}	γ_i^{I}	S_{min}^{I}	q_i^{II}	γ_i^{II}	S_{min}^{II}	q_i^{CP}	γ_i^{CP}	S_{min}^{CP}
120	−7	−7	**20**	−10	−10	**20**	−17	−17	**23**
100	−5	−2		−10	−20		−5	−22	
80	−15	−17	W_{min}^{I}	14	−6	W_{min}^{II}	−1	−23	W_{min}^{CP}
60	−3	−20		10	4		7	−16	
40	3	−17	**3**	5	9	**29**	8	−18	**15**

Max. potential savings between pinches = **17** Max. possible savings = **17**

curves, the approach of Ahmad and Hui (1991) is employed. Figure 5 shows the countercurrent profiles for the grand composite curves of the two plants. The grand composite curve of plant 1 has been inverted to be able to establish the maximum amount of direct heat transfer. The extent of the maximum possible savings is reached whenever the profiles coincide in a point, as shown. Unassisted cases are therefore readily tractable with the reported method.

Example of assisted cases

Table 2 presents the data corresponding to Example 2. A source interval is located in the region above the pinch of plant 2 (higher pinch) in plant 1. This source interval prevents plant 1 from receiving all the potential heat available to be transferred between pinches. However, a transfer of the necessary amount from plant 1 to plant 2 above the higher pinch allows maximum potential savings to be realized. Assisted cases below the two pinches are similar in nature and therefore examples are omitted. The combined pinch lies between pinches. This is a result of the fact that all the limitation for transfer between pinches can be completely removed. Figure 6 shows the cascade diagram solution after the integration is conducted.

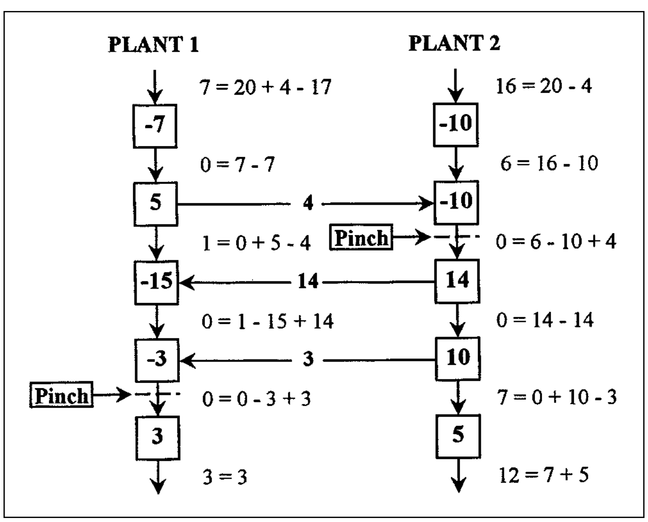

Figure 6. Cascade diagram solution for Example 2.

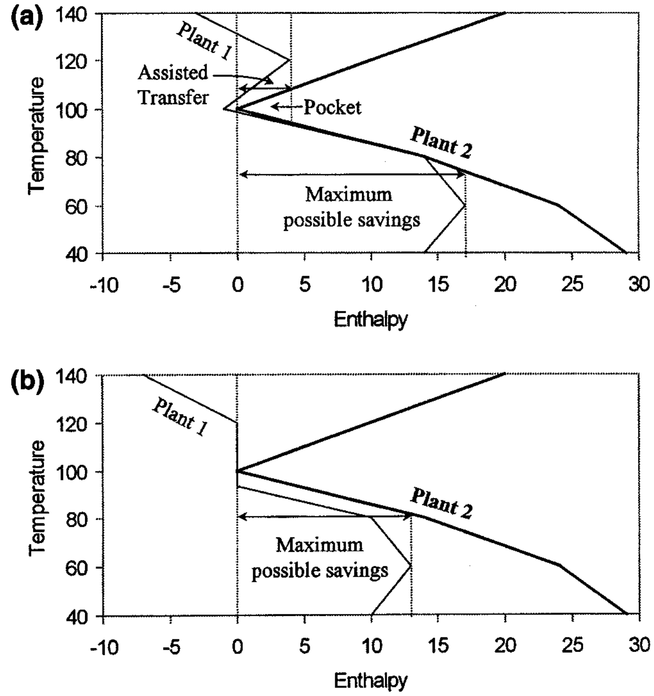

Figure 7. Countercurrent composite curve profiles for Example 2.

When a comparison with the method that uses grand composite curves is performed, the diagram of Figure 7a is obtained. The pocket present in the composite curve of plant 1 has not been removed, since it makes possible the assisted transfer to plant 2 and allows full transfer of heat between pinches. In the procedure introduced by Dhole and Linnhoff (1992) to indirectly integrate the total site through the utility system, pockets are eliminated prior to the construction of the site-source and site-sink profiles. Therefore, whenever the pockets are eliminated, the possibility of realizing maximum savings has been lost. This is illustrated in Figure 7b.

Table 3 presents the data for Example 3. A source interval in plant 1 is found in the region above the pinch of plant 2. Thus, this is an assisted case. However, a limit imposed by plant 2 arises in the heat that plant 1 can transfer above the higher pinch. Therefore, the limitation to obtain maximum potential savings cannot be totally removed. Figure 8 shows

Table 3. Example 3

Temp. Scale 140	Plant 1			Plant 2			Combined Plant		
	q_i^{I}	γ_i^{I}	S_{min}^{I}	q_i^{II}	γ_i^{II}	S_{min}^{II}	q_i^{CP}	γ_i^{CP}	S_{min}^{CP}
120	−18	−18	**20**	−19	−19	**20**	−37	−37	**37**
100	5	−13		−1	−20		4	−33	
80	−5	−18	W_{min}^{I}	10	−10	W_{min}^{II}	5	−28	W_{min}^{CP}
60	−2	−20		5	−5		3	−25	
40	2	−18	**2**	1	−4	**16**	3	−22	**15**

Max. potential savings between pinches = **7** Max. possible savings = **3**

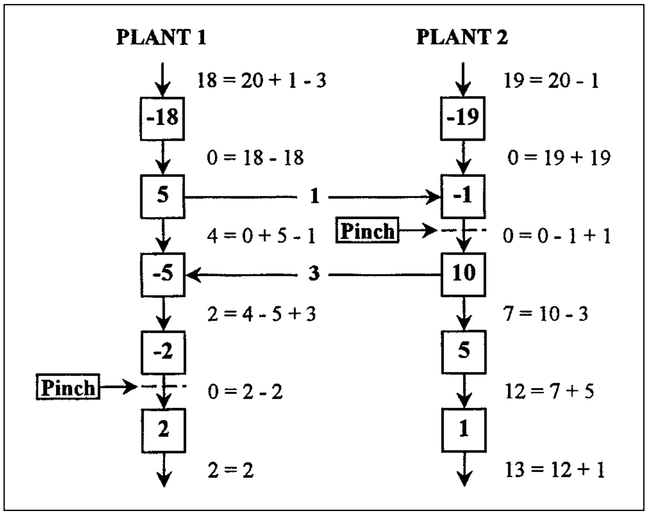

Figure 8. Cascade diagram solution for Example 3.

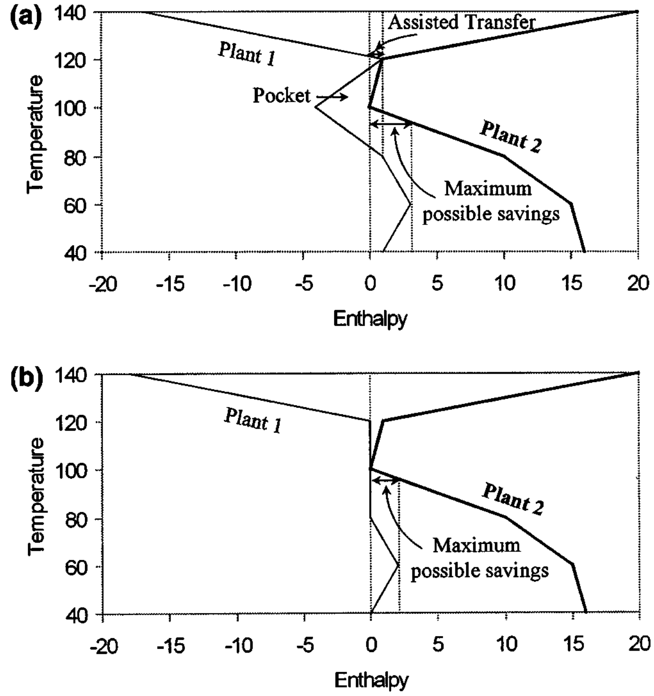

Figure 9. Countercurrent composite curve profiles for Example 3.

the cascade diagram solution after the integration is conducted.

Figure 9a shows the composite curves for this example. In this case, the presence of a pocket in plant 1 allows the partial removal of the limitations in the transfer between pinches. The elimination of this pocket prevents the realization of the maximum possible savings as it is shown in Figure 9b.

Targeting Model for Heat Integration

In this section, a model that allows the automatic determination of unassisted and assisted cases is presented. This model predicts the amount of heat that needs to be transferred in each interval to achieve maximum savings. Application to either direct or indirect integration is possible. In order to facilitate the computations, the temperature intervals are constructed using inlet and outlet temperatures of all streams from both plants (that is, $m^{\text{I}} = m^{\text{II}} = m$).

Maximum energy savings

Maximum savings that can be obtained by integration are computed subtracting the combined plant minimum heating utility from the summation of the individual plant's minimum heating utilities. To obtain the amount of heat that has to be transferred in each interval, a model is constructed where heat can be transferred independently within each interval (Figure 10). A single direction of heat transfer is allowed: from plant 2 to plant 1 between pinches and from plant 1 to plant 2 outside this region.

The task is now to determine what amount is transferred at each interval to achieve maximum savings. To do that, an LP model is proposed. Let $\hat{\delta}_0^{\text{I}}$ and $\hat{\delta}_0^{\text{II}}$ be the original minimum heating utility of plant 1 and plant 2, respectively, when no integration between plants is assumed. These values are $S_{\min}^{\text{I}}$ and $S_{\min}^{\text{II}}$, the results obtained by solving the LP transportation or transshipment models for each of the plants separately. In the same way, let $\hat{\delta}_m^{\text{I}}$ and $\hat{\delta}_m^{\text{II}}$ be the original cooling utilities ($W_{\min}^{\text{I}}$ and $W_{\min}^{\text{II}}$ values). Also let δ_i^{I} and δ_i^{II} be the new heat transferred between intervals after integration between plants is implemented. Finally, let q_i^E be the heat transferred between pinches in interval i from plant 2 to plant 1, and q_i^A and q_i^B the heat transferred in the inverse direction above the higher pinch and below the lower pinch, respectively.

The model that predicts the maximum possible energy savings that effectively occur between pinches Q_E, and the eventual minimum amount of heat Q_A and Q_B to be transferred

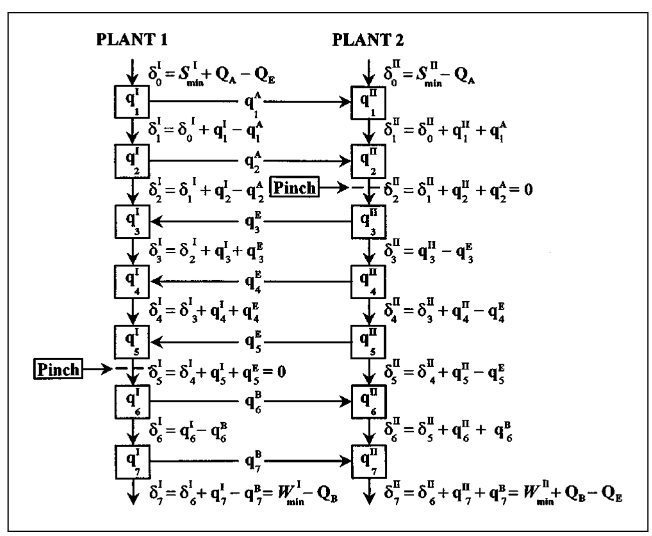

Figure 10. Splitting the heat transfer among intervals.

in the regions outside the one between pinches is

$$
\begin{aligned}
&\text{Min}\left(\delta_0^I + \delta_m^{II}\right)\\
&\text{s.t.}\\
&\delta_0^{I} = \hat{\delta}_0^{I} + Q_A - Q_E\\
&\delta_0^{II} = \hat{\delta}_0^{II} - Q_A\\
&\left.\begin{aligned}\delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} - q_i^{A}\\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} + q_i^{A}\end{aligned}\right\} \quad \forall i = 1, \ldots, p^{II}\\
&\left.\begin{aligned}\delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} + q_i^{E}\\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} - q_i^{E}\end{aligned}\right\} \quad \forall i = (p^{II}+1), \ldots, p^{I}\\
&\left.\begin{aligned}\delta_i^{I} &= \delta_{i-1}^{I} + q_i^{I} - q_i^{B}\\ \delta_i^{II} &= \delta_{i-1}^{II} + q_i^{II} + q_i^{B}\end{aligned}\right\} \quad \forall i = (p^{I}+1), \ldots, m\\
&\delta_m^{I} = \hat{\delta}_m^{I} - Q_B\\
&\delta_m^{II} = \hat{\delta}_m^{II} + Q_B - Q_E\\
&\delta_{p^{I}}^{I} = 0, \qquad \delta_{p^{II}}^{II} = 0\\
&\delta_i^{I},\ \delta_i^{II},\ q_i^{A},\ q_i^{E},\ q_i^{B} \geq 0.
\end{aligned} \tag{1}
$$

In this formulation p^{I} and p^{II} are the respective pinch point levels, and it is assumed $p^{I} < p^{II}$. The problem considers the conditions of minimum utility usage for both plants as the starting point. The objective function used needs some explanation. Minimizing the utility needed in plant 1 serves two purposes:

1. To reduce the utility in the amount transferred from plant 2 between pinches.
2. To make sure that the amount of heat transferred from plant 1 to plant 2 is strictly the minimum needed.

When a higher amount of heat than the minimum needed is transferred above pinches in the assisted case, the excess consists of a simple shift of utility from plant 2 to plant 1. Such shifting requires equipment and therefore represents additional investment without a benefit and should be avoided. The same result is obtained if one solves the problem minimizing the amount of cooling utility of plant 2. In this case, the transfer to plant 1 is maximized, while the transfer from plant 1 to plant 2 below both pinches is kept at its minimum necessary to assist in the savings. Finally, for each unit of heat transferred between plants, both values are reduced by the same amount simultaneously. This implies that independent reductions of these utilities are not possible. Hence, adding them to form the objective function of problem 1 is possible. A simple balance around plant 1 proves that the summation of the solutions (heat transferred amounts q_i^E) will represent the total possible amount of heat to be transferred between pinches Q_E.

Remark. The LP problem presented has degenerate solutions. Indeed, to transfer heat surplus from an interval in plant 2 to any interval in plant 1 between pinches, the heat can be transferred from plant 2 to plant 1 first and then transferred down, or transferred down in plant 2 first, and then transferred to plant 1 at a lower interval. The same situations occur in an inverse manner when the transfer takes place in any of the regions outside the region between pinches. Therefore, many different paths are available. This degeneracy is actually a flexibility that can be exploited later when a design is attempted.

The results from the preceding models can now be used as target values for models that will determine the heat-exchanger network needed to accomplish such savings. In particular, the knowledge of what are the intervals at which heat transfer from one plant to the other should take place (in addition to the direction of such transfer) is a useful input for these models. These models, which will be presented in a follow-up article will address the design of systems featuring the minimum number of exchanger units to accomplish a dual operation (with and without integration).

Indirect Heat Integration

The focus is now on the case of indirect heat integration by the use of intermediate fluid circuits. The design parameters for these circuits, namely flow rate and inlet and outlet temperatures, have to be calculated.

Shift of scales

When an intermediate fluid is used, new streams appear in each plant. Consider the region between pinches first. In plant 1, the intermediate fluid acts as a hot stream, whereas in plant 2, it acts as a cold stream. The temperature of the intermediate fluid leaving plant 1 (registered in its hot scale) should be equal to the starting temperature of the same fluid in plant 2, requiring the coincidence between the respective hot and cold scales. Thus, a shift consisting of moving the hot scale of plant 2 (and with it, the cold scale too) downward $\Delta T_{\min}$ degrees in the region below its pinch is performed. Now consider the possibility of assisted cases. In the region above the higher pinch and below the lower pinch, the fluid circulates in the inverse direction than between pinches. Therefore, a match between the cold scale of plant 1 and the hot scale of plant 2 is required in these two regions. To accomplish this, the hot scale of plant 2 (and with it, the cold scale) is shifted upward $\Delta T_{\min}$ degrees in the zone above its pinch. Similarly, in the region below the lower pinch, a shift of the hot scale of plant 1 (and with it, the cold scale) downward is needed. However, the hot scale of plant 2 was already shifted by $\Delta T_{\min}$. Therefore, a shift of $2\Delta T_{\min}$ degrees downward of the hot scale of plant 1 (and, with it, its cold scale) has to be performed. Finally, as in the direct integration case, the temperature intervals are constructed using inlet and outlet temperatures of all streams of both plants.

As a result of these temperature shifts, smaller savings than in the direct integration case may be achieved. If the use of intermediate fluids is not mandatory (due to safety or other considerations), then this reduction in savings potential may or may not be compensated by the reduction in piping, pumping, and/or compression costs.

Summarizing, the scale shifts required are:

1. A shift of both hot and cold scales downward by $\Delta T_{\min}$ degrees in plant 2 below its pinch.

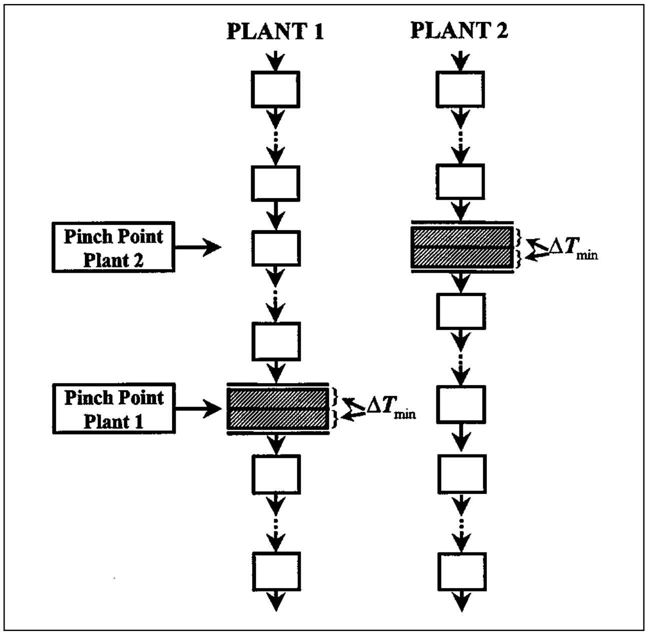

Figure 11. Shift of scales to allow the use of intermediate fluids.

2. A shift of both hot and cold scales upward by $\Delta T_{\min}$ degrees in plant 2 above its pinch.

3. A shift of both hot and cold scales downward by $2\Delta T_{\min}$ degrees in plant 1 below its pinch.

These shifts create gaps in the scales, as depicted in Figure 11.

Maximum energy savings

The maximum savings that can be obtained by integration can be computed by subtracting the combined plant minimum heating utility using the shifted scales from the summation of the individual plant's minimum heating utilities. Next, to establish the regions in which heat transfer should be made to accomplish the overall target, problem 1 is solved.

Remark. The solution to problem 1 can be implemented in practice. Indeed, a circuit can be established for each interval that has a nonzero heat transfer q_i^E, q_i^A, or q_i^B. However, one is interested in performing the transfer with the smallest amount of circuits possible. The issue is investigated in the following sections.

Feasibility of a single-fluid circuit

Even though the target value for Q_E can always be transferred between pinches and the eventual target amounts Q_A and Q_B can always be transferred outside this region, the question is whether these transfers can be achieved with a single circuit in each region. Figure 12a shows the unassisted case with m_E intervals within the region between pinch temperatures $T_{p^{II}}$ and T_{p^I}. The assisted cases are shown in Figure 12b.

In the region between pinches, the heat released in each interval from plant 2 to the intermediate fluid does not have to be the same as the heat released by the fluid to plant 1 in the same interval. Likewise, in the region above the higher pinch and the region below the lower pinch, the heat released from plant 1 in the same interval does not have to be the same as the heat received by plant 2. A generalization for any of the regions comes next. The use of separate variables for the heat transferred to and from the intermediate fluid are defined as follows: q_i^{FC} for the heat received by the fluid in interval i, and q_i^{FH} for the heat released by the fluid in interval i. The total heat transferred Q_F is already given by the targeting procedure, that is,

$$Q_F = \sum_{i=k^F}^{k^F+m_K} q_i^{FC} = \sum_{i=k^F}^{k^F+m_K} q_i^{FH}, \tag{2}$$

where k^F is the first interval where the transfer between plants takes place and m_K represents the number of intervals covered by a circuit in any of the regions.

Temperature constraints

Let us first explore the second law constraints regarding q_i^{FC} and q_i^{FH}. A circuit covering m_E intervals between pinches will receive heat from plant 2 and will deliver it to plant 1. In the assisted cases, a circuit covering either m_A or m_B intervals will perform the inverse task, carrying heat from plant 1 to plant 2. Therefore, this can be generalized for a set of m_K intervals. The plant that is providing the heat is considered the "heat-source plant," while the one receiving that heat is considered the "heat-sink plant." The following constraints prevent the temperature of the fluid from going

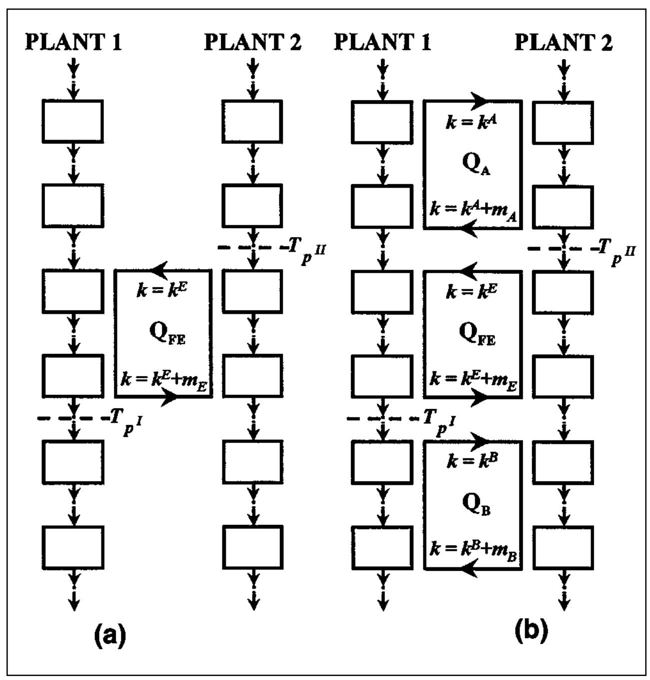

Figure 12. Circuits of intermediate fluids in unassisted and assisted cases (gaps are omitted for simplicity).

higher than the interval temperature T_{k-1} in the heat-source plant:

$$F\left(T_{k-1} - T_0^{FC}\right) \geq \sum_{i=k}^{k^F+m_K} q_i^{FC} \quad \forall k = \left(k^F + m_K\right), \ldots, \left(k^F + 1\right) \tag{3}$$

$$F\left(T_{k^F-1}^{FC} - T_0^{FC}\right) = Q_F. \tag{4}$$

In the heat-sink plant, similar constraints are introduced to prevent the temperature of the fluid going lower than the interval temperature T_k:

$$F\left(T_0^{FH} - T_k\right) \geq \sum_{i=k^F}^{k} q_i^{FH} \quad \forall k = k^F, \ldots, \left(k^F + m_K - 1\right) \tag{5}$$

$$F\left(T_0^{FH} - T_{k^F+m_K}^{FH}\right) = Q_F. \tag{6}$$

In order to assure a closed circuit, it should be noticed that the following has to be verified:

$$T_0^{FC} = T_{k^F+m_K}^{FH} \tag{7}$$

$$T_0^{FH} = T_{k^F-1}^{FC}. \tag{8}$$

Let us now examine what values the initial temperatures of the intermediate fluid can take. First, note that to guarantee feasibility of heat transfer, T_0^{FC} and T_0^{FH} are equal to T_{k-1} and T_k, respectively, for some k (that is, they are confined to be end-interval temperatures). Now consider the case where the heat-source plant does not have any heat demand in the first set of $(k^+ - k^F + 1)$ intervals, that is, $q_i^{FH} = 0$, $\forall i = k^F$, $\ldots$, k^+. Then by increasing the flow rate of the intermediate fluid and without limitations in the transfer of heat from the heat-source plant, a new solution with an upper temperature smaller than the one considered initially is possible. If this is the case, the heat-source plant will only be transferring heat to the intermediate fluid at temperatures lower than T_{k^+}. To find such a solution, heat can be cascaded down from the first $(k^+ - k^F + 1)$ intervals in the heat-source plant. The values of q_i^{FC} can be transformed to a new set $\hat{q}_i^{FC}$ as follows:

$$\hat{q}_i^{FC} = 0 \quad \forall i = k^F, \ldots, k^+ \tag{9}$$

$$\hat{q}_{k^++1}^{FC} = q_{k^++1}^{FC} + \sum_{i=k^F}^{k^+} q_i^{FC} \tag{10}$$

$$\hat{q}_i^{FC} = q_i^{FC} \quad \forall i = k^+ + 2, \ldots, \left(k^F + m_K\right), \tag{11}$$

where $\hat{q}_i^{FC}$ is another degenerate solution. This solution allows the circuit to be established between the intervals $k^+ + 1$ and $(k^F + m_K)$. A similar argument can be made for the case where the last intervals in plant 2 do not transfer heat to the intermediate fluid.

Thus, one can assume without loss of generality that the initial temperatures of the intermediate fluid in the heat-source and sink plants are

$$T_0^{FC} = T_{k^F+m_K}^{FH} = T_{k^F+m_K} \tag{12}$$

$$T_0^{FH} = T_{k^F-1}^{FC} = T_{k^F-1}. \tag{13}$$

With these equalities, the set of Eqs. 3–6 become

$$F\left(T_{k^F-1} - T_k\right) \geq \sum_{i=k^F}^{k} q_i^{FH} \quad \forall k = k^F, \ldots, \left(k^F + m_K - 1\right) \tag{14}$$

$$F\left(T_k - T_{k^F+m_K}\right) \geq \sum_{i=k}^{k^F+m_K} q_i^{FC} \quad \forall k = \left(k^F + 1, \ldots, m_K\right) \tag{15}$$

$$F\left(T_{k^F-1} - T_{k^F+m_K}\right) = Q_F \tag{16}$$

Equations 14–16 constitute a feasibility test for a single circuit transferring the heat Q_F between plants. The flow rate F can be calculated using Eq. 16. This value then can be replaced in Eqs. 14 and 15. If any of these equations are not satisfied, then a circuit between T_{k^F-1} and $T_{k^F+m_K}$ cannot transfer the maximum amount, but perhaps some smaller value.

Candidate heat-transfer sets

The flexibility at hand for defining the general variables q_i^{FH} and q_i^{FC} is explored by an adjusted heat-cascaded diagram (Figure 13). The target values Q_E, Q_A, and Q_B are added and subtracted in the three defined zones of the cascade. This accounts for the supplies or demand each of the

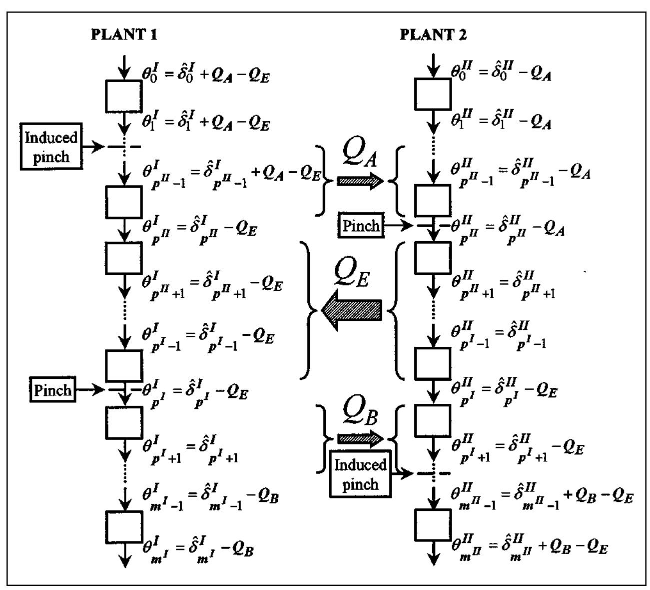

Figure 13. Adjusted heat-cascaded diagram (gaps are omitted for simplicity).

zones will experience when the respective single-circuit candidates are considered. The values obtained for the different intervals are not realistic heat-transfer amounts since some of them are negative, but rather a calculation aid. Moreover, because of these operations the adjusted heat-cascaded diagram of the heat-source plant may exhibit an induced pinch at each region where a circuit can be installed. In this instance, the transfer will only occur in the subzone delimited by the real and induced pinches.

In the unassisted case, the solutions of Eq. 1 satisfy the following relations:

$$\delta_k^{\mathrm{I}} = \hat{\delta}_{p^{\mathrm{II}}}^{\mathrm{I}} - Q_E + \sum_{i=p^{\mathrm{II}}+1}^{k} \left(q_i^{\mathrm{I}} + q_i^{E} \right) \geq 0 \qquad \forall k = \left(p^{\mathrm{II}}+1 \right), \ldots, p^{\mathrm{I}} \tag{17}$$

$$\delta_k^{\mathrm{II}} = \sum_{i=p^{\mathrm{II}}+1}^{k} \left(q_i^{\mathrm{II}} - q_i^{E} \right) \geq 0 \qquad \forall k = \left(p^{\mathrm{II}}+1 \right), \ldots, p^{\mathrm{I}}. \tag{18}$$

The same set of equations can be written in terms of q_i^{EH} and q_i^{EC}:

$$\delta_k^{\mathrm{I}} = \hat{\delta}_{p^{\mathrm{II}}}^{\mathrm{I}} - Q_E + \sum_{i=p^{\mathrm{II}}+1}^{k} \left(q_i^{\mathrm{I}} + q_i^{EH} \right) \geq 0 \qquad \forall k = \left(p^{\mathrm{II}}+1 \right), \ldots, p^{\mathrm{I}} \tag{19}$$

$$\delta_k^{\mathrm{II}} = \sum_{i=p^{\mathrm{II}}+1}^{k} \left(q_i^{\mathrm{II}} - q_i^{EC} \right) \geq 0 \qquad \forall k = \left(p^{\mathrm{II}}+1 \right), \ldots, p^{\mathrm{I}}. \tag{20}$$

Similarly, for the assisted cases the relations for the upper and lower zones are:

$$\delta_k^{\mathrm{I}} = \hat{\delta}_0^{\mathrm{I}} + Q_A - Q_E + \sum_{i=1}^{k} \left(q_i^{\mathrm{I}} - q_i^{AC} \right) \geq 0 \qquad \forall k = 1, \ldots, p^{\mathrm{II}} \tag{21}$$

$$\delta_k^{\mathrm{II}} = \hat{\delta}_0^{\mathrm{II}} - Q_A + \sum_{i=1}^{k} \left(q_i^{\mathrm{II}} + q_i^{AH} \right) \geq 0 \qquad \forall k = 1, \ldots, p^{\mathrm{II}} \tag{22}$$

$$\delta_k^{\mathrm{I}} = \sum_{i=p^{\mathrm{I}}+1}^{k} \left(q_i^{\mathrm{I}} - q_i^{BC} \right) \geq 0 \qquad \forall k = \left(p^{\mathrm{I}}+1 \right), \ldots, m \tag{23}$$

$$\delta_k^{\mathrm{II}} = \hat{\delta}_{p^{\mathrm{I}}}^{\mathrm{I}} - Q_E + \sum_{i=p^{\mathrm{II}}+1}^{k} \left(q_i^{\mathrm{II}} + q_i^{BH} \right) \geq 0 \qquad \forall k = \left(p^{\mathrm{I}}+1 \right), \ldots, m. \tag{24}$$

Thus, any nonnegative set of generalized values q_i^{FH} and q_i^{FC} that satisfies Eqs. 19–20, 21–22, or 23–24, and also the balance equation (Eq. 2) is an acceptable candidate for a single circuit. If in addition, Eqs. 14–16 are satisfied, the candidate set will be a feasible single-circuit solution in any of the zones. A few generalized candidate sets are presented next.

One candidate set is given by the solution of problem 1, that is,

$$q_i^{FH} = q_i^{FC} = q_i^{F}. \tag{25}$$

Other sets can be found by making use of degeneracy. In particular, one can choose a set that prioritizes the heat transfer to the intermediate fluid over the heat transfer to the interval below in the source plant. This solution is called the *higher-circuit* solution, because the circuit starts and ends at the higher possible intervals. A lower circuit solution is presented later. The maximization of the heat delivered to the intermediate fluid is the purpose of constructing a higher circuit solution.

Therefore, to establish the maximum amount of heat that each interval can provide to the intermediate fluid, the deficit of heat in the intervals below it need to be taken into account. The heat availability ω_k^H at each interval is then defined as follows: Let $\lambda_k^H = \mathrm{Min}\{q_k^H, 0\}$ and $\sigma_k{}^H = \mathrm{Max}\{q_k^H, 0\}$ $\forall k = k^F+1, \ldots, k^F + m_k$. In addition, let $\lambda_{k^F}^H = 0$, $\sigma_{k^F}^H = q_{k^F}^H + \hat{\delta}_{k^F-1}^H$. Thus, the availability ω_k^H is given by

$$z_{k^F+m_K}^H = \begin{cases} \lambda_{k^F+m_K}^H + \sigma_{k^F+m_K}^H & q_{k^F+m_K}^H + \lambda_{k^F+m_K}^H < 0 \\ 0 & q_{k^F+m_K}^H + \lambda_{k^F+m_K}^H \geq 0 \end{cases} \tag{26}$$

$$z_k^H = \begin{cases} z_{k+1}^H + \lambda_k^H + \sigma_k^H & q_k^H + z_{k+1}^H + \lambda_k^H < 0 \\ 0 & q_k^H + z_{k+1}^H + \lambda_k^H \geq 0 \end{cases} \qquad \forall k = k^F+1, \ldots, k^F + m_K - 1. \tag{27}$$

$$z_{k^F}^H = \begin{cases} z_{k^F+1}^H + \lambda_{k^F}^H + \sigma_{k^F}^H & q_{k^F}^H + z_{k^F+1}^H + \lambda_{k^F}^H < 0 \\ 0 & q_{k^F}^H + z_{k^F+1}^H + \lambda_{k^F}^H \geq 0 \end{cases} \tag{28}$$

$$\omega_k^H = \mathrm{Max}\left\{ q_k^H + z_{k+1}^H + \lambda_k^H, 0 \right\} \qquad \forall k = k^F, \ldots, k^F + m_K. \tag{29}$$

In these equations, z_k^H is an auxiliary variable that helps determine the amount of cumulative demand (from the bottom) at every interval. To illustrate this, consider first the situation depicted in Table 4 for which $\hat{\delta}_{k^F-1}^H = 0$, that is,

Table 4. Determination of Heat Availability

Interval	q_i^H	λ_k^H	$\sigma_k{}^H$	$q_k^H + z_{k+1}^H + \lambda_k^H$	z_k^H	ω_k^H
k^F	12	0	12	8	0	8
k^F+1	−1	−1	0	−5	−4	0
k^F+2	−1	−1	0	−4	−3	0
k^F+3	15	0	15	−2	−2	0
k^F+4	−15	−15	0	−32	−17	0
k^F+5	−2	−2	0	−4	−2	0
k^F+6	20	0	20	20	0	20

either a higher circuit between pinches or a circuit above all pinches with an induced pinch.

Under such conditions, all resulting heat flows cascaded down are positive. On the source side, the higher-circuit solution is given by

$$q_{k^F}^{FC} = \text{Min}\left\{\omega_{k^F}^{H}, Q_F\right\} \tag{30}$$

$$q_k^{FC} = \text{Min}\left\{\omega_k^{H}, Q_F - \sum_{i=k^F}^{k-1} q_i^{FC}\right\} \quad \forall k = k^F+1, \ldots, k^F + m_K. \tag{31}$$

Equation 30 states that at the first interval, all the heat available will be used provided it is lower than the overall maximum. Equation 31 states that at every interval, the maximum that can be transferred is the surplus. In turn, for the sink plant, the higher solution is

$$q_{k^F}^{FH} = Q_F \tag{32}$$

$$q_k^{FH} = 0 \quad \forall k = k^F+1, \ldots, k^F + m_K. \tag{33}$$

This means that if the sink plant can transfer all the maximum possible heat Q_F in the first interval of transfer k^F, then the higher-circuit solution will consist of this interval only.

At the other extreme, we have a solution that maximizes the transfer of heat to the interval below in the heat-source plant, minimizing the transfer to the intermediate fluid. This solution is called the *lower-circuit* solution because it starts and ends at the lowest intervals possible. In such case, the solutions for the sink and the source plants are somewhat related. Indeed, by transferring heat to lower intervals in the source plant, one must make sure that the plant does not need the heat at the same interval. The adjusted cascaded heat values already account for this. Then, the lower-circuit solution for the sink plant is given by

$$q_{k^F}^{FH} = \text{Max}\left\{-\theta_{k^F}^{C}, 0\right\} \tag{34}$$

$$q_k^{FH} = \text{Max}\left\{-\theta_k^{C} - \sum_{i=k^F}^{k-1} q_i^{FH}, 0\right\}, \quad \forall k = k^F+1, \ldots, k^F + m_K. \tag{35}$$

Now let k^+ be the first interval with nonzero heat transferred from the intermediate fluid to the sink plant, that is, k^+ is such that $q_k^{FH} = 0, \forall k = k^F, \ldots, (k^+ - 1)$; $q_{k^+}^{FH} \neq 0$. Next, all the surplus heat in the source plant can be transferred down until interval k^+ is reached. From then on, only the minimum amount of heat should be transferred to the intermediate fluid. This minimum should be at least equal to q_i^{FH} to guarantee that temperature constraints have a chance of being satisfied. Thus, the lower-circuit solution for the source plant is

$$q_k^{FC} = 0 \quad \forall k = k^F, \ldots, (k^+ - 1) \tag{36}$$

$$q_k^{FC} = q_k^{FH} \quad \forall k = k^+, \ldots, k^F + m_K. \tag{37}$$

By construction, no one-circuit solution can:

1. Start at a lower interval;
2. Transfer less heat from the intermediate fluid to the sink plant at any interval defined by the lower solution.

As a result of the calculation of the cumulative heat demands, a limit for the starting point of the unique circuit is established. In view of the preceding, a second test for the feasibility of a single circuit transferring the maximum savings Q_F in any of the regions consists of constructing the higher or lower solution and checking if the following equations are satisfied:

$$Q_F \frac{\left(T_{k^F-1} - T_k\right)}{\left(T_{k^F-1} - T_{k^F+m_K}\right)} \geq \sum_{i=k^F}^{k} q_i^{FH} \quad \forall k = k^F, \ldots, \left(k^F + m_K - 1\right) \tag{38}$$

$$Q_F \frac{\left(T_k - T_{k^F+m_K}\right)}{\left(T_{k^F-1} - T_{k^F+m_K}\right)} \geq \sum_{i=k}^{k^F+m_K} q_i^{EC} \quad \forall k = \left(k^F + m_K\right), \ldots, \left(k^F + 1\right). \tag{39}$$

These equations have been obtained by substituting Eq. 16 in Eqs. 14 and 15.

It should be noted that the lower solution obtained by the preceding procedure is not always feasible. Some other lower solutions might exist, not necessarily covering the last intervals of the region between pinches, but covering a region that ends somewhere above it.

Example 4

In this example, Test Case #2 from Linnhoff and Hindmarsh (1983) is plant 1 and problem 4sp1 is plant 2. The data for the separate plants are shown in Table 5. Note that $\Delta T_{\min}$ for plant 1 is 20°C, while $\Delta T_{\min}$ for plant 2 is 10°C. Pinch temperatures and minimum utility consumption for each of the plants are shown in Table 6.

Direct Integration Solution. Table 7 shows the results of the pinch analysis. The interval between pinches goes from

Table 5. Data for Example 4

Streams	F (kW/°C)	T_s (°C)	T_t (°C)	Q (kW)
Test Case #2				
H1 (Hot)	2.0	150	60	180.0
C2 (Cold)	2.5	20	125	262.5
H3 (Hot)	8.0	90	60	240.0
C4 (Cold)	3.0	25	100	225.0
S (Stream)	—	270	270	107.5
CW (Water)	0.9	38	82	40.0
	$\Delta T_{\min} = 20$°C			
Problem 4sp1				
C1 (Cold)	7.62	60	160	762
H2 (Hot)	8.79	160	93	589
C3 (Cold)	6.08	116	260	876
H4 (Hot)	10.55	249	138	1171
S (Steam)	—	270	270	128
CW (Water)	5.68	38	82	250
	$\Delta T_{\min} = 10$°C			

Table 6. Individual Plant Pinch Analysis for Example 4

Problem	Pinch Temp. (°C)	Heating Utility (kW)	Cooling Utility (kW)
Test case #2	90	107.5	40.0
4sp1	249	128.0	250.0

90°C to 249°C. After considering all streams in a single set, the resulting combined pinch is located at 249°C (upper limit of the interval between pinches). This is the consequence of the large availability of heat to transfer that plant 2 has in all the intervals between pinches. This amount of heat is sufficient to supply the entire demand of plant 1. Therefore, the maximum possible heat savings for the direct integration are the original minimum utility of plant 1, that is 107.5 kW. This is also the result obtained by solving problem 1.

Indirect Integration Solutions. After the hot temperature scale in plant 2 is shifted down 10°C, the interval between pinches is from 239°C to 90°C. Table 8 shows the pinch analysis for the indirect integration. The combined pinch then results at the upper bound (239°C). Fewer intervals than in the case of direct integration are found, since some of the extreme temperatures now coincide due to the shift. The solution of problem 1 is $Q_F = 107.5$ kW, which is equal to the maximum possible savings that can be obtained either with direct or indirect integration. Therefore, in this case the shift does not have any effect in reducing the amount that can be transferred between pinches. Table 8 shows that there is no demand in the upper intervals of plant 1, and the shift does not appreciably decrease the large availability of heat in plant 2 in the region between pinches.

An implementation of this indirect integration follows. The test of feasibility for a single circuit is applied first to a circuit covering all the intervals between pinches. This solution is feasible, and Table 9 shows the values obtained for the parameters (ending temperatures and rate-heat-capacity product). The higher-circuit solution is shown in Table 10. The position of the resulting circuit is shown in Figure 14. Note that Eqs. 38 and 39 are satisfied. Moreover, the intermediate solutions between the circuit spanning all intervals and the highest possible circuit are feasible.

Finally, the lower-circuit solution is presented in Table 11, and its position is shown in Figure 15. The intermediate circuits between the circuit spanning all intervals and the lowest possible circuit are proven feasible. Other solutions can be found each time that a certain amount of heat could be cascaded and the one-circuit solutions that result are feasible. However, no solution will be able to start below the limit established by the lower solution. In this sense, the problem has a large finite number of possible solutions that require further analysis, taking into account the resulting heat-exchanger network and the economic aspects.

Table 7. Pinch Analysis for Direct Integration in Example 4

	Test Case #2			4sp1			Combined Plant		
T (°C)	q_i^I (kW)	γ_i^I (kW)	S_{min}^I (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S_{min}^{II} (kW)	q_i^C (kW)	γ_i^C (kW)	S_{min}^{CP} (kW)
270	0	0	**107.5**	−127.8	−127.8	**127.8**	−127.8	−127.8	**127.8**
249	0	0		353.1	225.3		353.1	225.3	
170	0	0		−31.5	193.8		−31.5	193.8	
160	0	0		56.4	250.2		56.4	250.2	
150	10.0	10.0		28.2	278.4		38.2	288.4	
145	−3.5	6.5		39.5	317.9		36.0	324.4	
138	−6.0	0.5		−58.9	259.0		−64.9	259.5	
126	−3.0	−2.5		7.0	266.0		4.0	263.5	
120	−94.5	−97.0		31.6	297.6		−62.9	200.6	
93	−10.5	−107.5		−22.9	274.7		−33.4	167.2	
90	90.0	−17.5	W_{min}^I (kW)	−152.4	122.3	W_{min}^{II} (kW)	−62.4	104.8	W_{min}^{CP} (kW)
70	45.0	27.5		0	122.3		45.0	149.8	
60	−82.5	−55.0		0	122.3		−82.5	67.3	
45	−12.5	−67.5	**40.0**	0	122.3	**250.0**	−12.5	54.8	**182.5**

Table 8. Pinch Analysis for Indirect Integration in Example 4

	Test Case #2			4sp1			Combined Plant		
T (°C)	q_i^I (kW)	γ_i^I (kW)	S_{min}^I (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S_{min}^{II} (kW)	q_i^C (kW)	γ_i^C (kW)	S_{min}^{CP} (kW)
260	0	0	**107.5**	−127.8	−127.8	**127.8**	−127.8	−127.8	**127.8**
239	0	0		353.1	225.3		353.1	225.3	
160	0	0		−31.5	193.8		−31.5	193.8	
150	10.0	10.0		28.2	222.0		38.2	232.0	
145	−8.5	1.5		95.8	317.8		87.3	319.3	
128	−4.0	−2.5		−39.3	278.5		−43.3	276.0	
120	−14.0	−16.5		−19.6	258.9		−33.6	242.4	
116	−91.0	−107.5		30.4	289.3		−60.6	181.8	
90	31.5	−76.0	W_{min}^I (kW)	8.2	297.5	W_{min}^{II} (kW)	39.7	221.5	W_{min}^{CP} (kW)
83	103.5	27.5		−175.3	122.2		−71.8	149.7	
60	−82.5	−55.0		0	122.2		−82.5	67.2	
45	−12.5	−67.5	**40.0**	0	122.2	**250.0**	−12.5	54.7	**182.5**

Table 9. Some of the Indirect Solutions to Example 4

Solution	No. of Intervals	T_{up} (°C)	T_{down} (°C)	F (kW/°C)
All intervals	7	239	90	0.721
Lower circuit	5	150	90	1.792
Higher circuit	2	239	150	1.208

Table 10. "Higher Circuit" Solution to Example 4

$Q_E = 107.5$ kW	Test Case #2		4sp1		
Interval	q_k^I (kW)	q_k^{EH} (kW)	q_k^{II} (kW)	ω_k^{II} (kW)	q_k^{EC} (kW)
$p^{II}+1=2$	0	107.5	353.1	321.6	107.5
$p^{II}+2=3$	0	0	−31.5	0	0
$p^{II}+3=4$	10	0	28.2	28.2	0
$p^{II}+4=5$	−8.5	0	95.8	36.9	0
$p^{II}+5=6$	−4.0	0	−39.3	0	0
$p^{II}+6=7$	−14.0	0	−19.6	0	0
$p^{II}+7=8$	−91.0	0	30.4	30.4	0

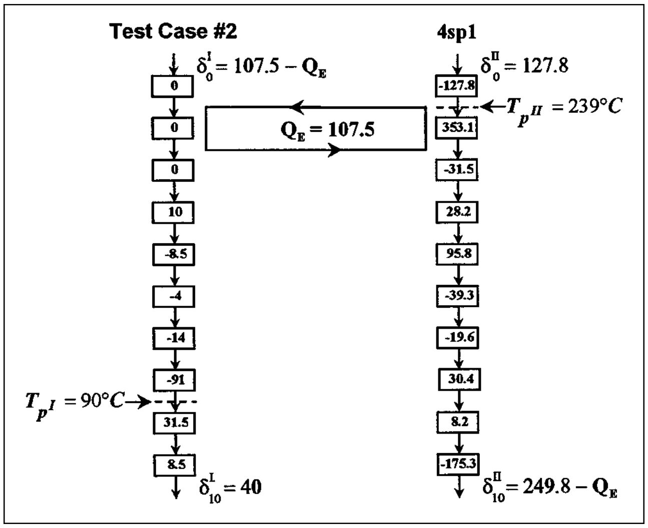

Figure 14. Higher single-circuit solution for Example 4.

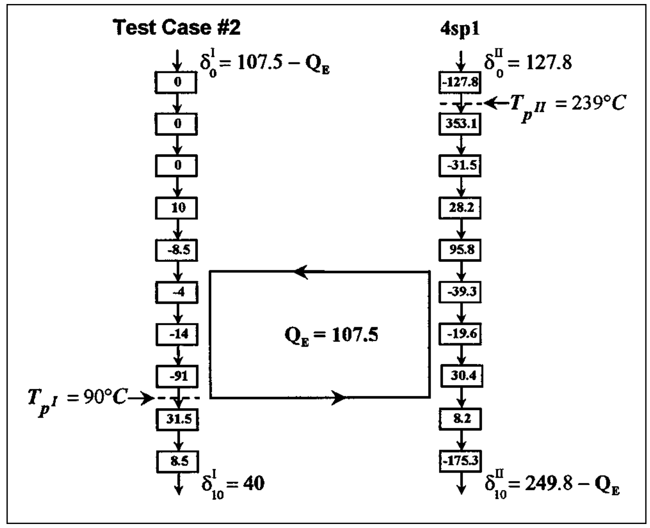

Figure 15. Lower single-circuit solution for Example 4.

Example 5

In this case, an example taken from Trivedi (1988) is plant 1 and example 1 from Ciric and Floudas (1991) is plant 2. The data for the separate plants are shown in Table 12. Pinch temperatures and minimum utility consumption for each of the plants are shown in Table 13.

Direct Integration Solution. Table 14 shows the results of the pinch analysis. The interval between pinches goes from 160°C to 200°C. After considering all streams in a single set, the resulting combined pinch is located at 200°C (upper limit of the interval between pinches). The maximum possible savings for the direct integration are 104.4 kW. Solving problem 1 gives the targeting values of the heat to be transferred in each of the zones. A minimum of 52.9 kW (Q_A) has to be transferred in the zone above both pinches in order to attain the maximum possible savings.

Indirect Integration Solutions. In this case, the hot temperature scale in plant 2 below its pinch is shifted down 20°C, while above the pinch the same scale is shifted up 20°C. A gap of 40°C is then created in plant 2, and no integration is possible in this zone. The interval between pinches is from 160°C to 180°C (hot scale of plant 1). Table 15 shows the pinch analysis for the indirect integration. Again, the combined pinch is at the upper bound of this interval (180°C).

Table 11. "Lower Circuit" Solution to Example 4

$Q_E = 107.5$ kW	Test Case #2			4sp1	
Interval	q_k^{I} (kW)	θ_k^{I} (kW)	q_k^{EH} (kW)	q_k^{II} (kW)	q_k^{EC} (kW)
$p^{\mathrm{II}}+1=2$	0	0	0	353.1	0
$p^{\mathrm{II}}+2=3$	0	0	0	−31.5	0
$p^{\mathrm{II}}+3=4$	10	10	0	28.2	0
$p^{\mathrm{II}}+4=5$	−8.5	1.5	0	95.8	0
$p^{\mathrm{II}}+5=6$	−4.0	−2.5	2.5	−39.3	2.5
$p^{\mathrm{II}}+6=7$	−14.0	−16.5	14.0	−19.6	14.0
$p^{\mathrm{II}}+7=8$	−91.0	−107.5	91.0	30.4	91.0

The maximum possible savings from indirect integration, 65.2 kW, are 38% lower than the savings from direct integration. Solving problem 1 gives the targeting values of the heat to be transferred in each of the zones. A minimum of 13.7 kW (Q_A) is to be transferred in the zone above both pinches to attain the maximum possible savings. This represents 26% of the amount to be transferred in the direct case.

Table 12. Data for Example 5

Streams	F (kW/°C)	T_s (°C)	T_t (°C)	Q (kW)
Trivedi				
H1 (Hot)	7.032	160	110	351.6
H2 (Hot)	8.44	249	138	936.8
H3 (Hot)	11.816	227	106	1,429.7
H4 (Hot)	7.0	271	146	875.0
C1 (Cold)	9.144	96	160	585.2
C2 (Cold)	7.296	115	217	744.2
C3 (Cold)	18	140	250	1,980.0
S (Steam)	—	300	300	404.8
CW (Water)	34.43	70	90	688.6
	$\Delta T_{\min} = 20$°C			
Ciric and Floudas				
H5 (Hot)	10	300	200	1,000
H6 (Hot)	120	200	100	12,000
C4 (Cold)	15	70	270	3,000
C5 (Cold)	25	70	190	3,000
C6 (Cold)	50	70	180	5,500
S (Steam)	—	300	300	600
CW (Water)	105	70	90	2,100
	$\Delta T_{\min} = 20$°C			

Table 13. Individual Plant Pinch Analysis for Example 5

Problem	Pinch Temp. (°C)	Heating Utility (kW)	Cooling Utility (kW)
Trivedi	160	404.8	688.6
Ciric and Floudas	200	600	2,100

Table 14. Pinch Analysis for Direct Integration in Example 5

	Trivedi			Ciric and Floudas			Combined Plant		
T (°C)	q_i^{I} (kW)	γ_i^{I} (kW)	S_{min}^{I} (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S_{min}^{II} (kW)	q_i^{C} (kW)	γ_i^{C} (kW)	S_{min}^{CP} (kW)
300	0	0	**404.8**	100.0	100.0	**600.0**	100.0	100.0	**900.4**
290	0	0		−95.0	5.0		−95.0	5.0	
271	7.0	7		−5.0	0.0		2.0	7.0	
270	−231.0	−224		−105.0	−105.0		−336.0	−329.0	
249	−3.07	−254.7		−60.0	−165.0		−90.7	−419.7	
237	−98.6	−353.3		−50.0	−215.0		−148.6	−568.3	
227	33.3	−320.0		−85.0	−300.0		−51.7	−620.0	
210	19.6	−300.4		−300.0	−600.0		−280.4	−900.4	
200	39.2	−261.2		600.0	0.0		639.2	−261.2	
180	−143.7	−404.8		600.0	600.0		456.3	195.2	
160	249.9	−155.0		420.0	1,020.0		669.9	865.0	
146	86.8	−68.2		240.0	1,260.0		326.8	1,191.8	
138	7.2	−61.0		90.0	1,350.0		97.2	1,289.0	
135	184.4	123.4		570.0	1,920.0		754.4	2,043.4	
116	113.1	236.5	W_{min}^{I}	180.0	2,100.0	W_{min}^{II}	293.1	2,336.5	W_{min}^{CP}
110	47.3	283.8	(kW)	120.0	2,220.0	(kW)	167.3	2,503.8	(kW)
106	0	283.8		180.0	2,400.0		180.0	2,683.8	
100	0	283.8	**688.6**	−900.0	1,500.0	**2,100.0**	−900.0	1,783.8	**2,684.1**

An implementation of this indirect integration follows. Table 16 shows the adjusted cascade heats. Only intervals above plant 1 pinch are shown since these intervals include the two zones of interest (upper and between pinches). Since an induced pinch appears in plant 1, a single interval is left for the transfer of the amount Q_A. In this case, a coincidence in the higher and lower circuits is found. A transfer of 13.7 kW is required in the upper zone. Once this amount is transferred, a circuit is established in the interval between pinches. This circuit will transfer 65.3 kW. Then, the solution for the indirect integration in Example 5 is shown in Figure 16.

Use of Composite Curves. For comparison, the method that uses countercurrent profiles for the grand composite curves is also applied to the direct integration of the plants (Figure 17a). A pocket in plant 1 allows the transfer of the required amount of heat above pinches that makes possible

Table 15. Pinch Analysis for Indirect Integration in Example 5

	Trivedi			Ciric and Floudas			Combined Plant		
T (°C)	q_i^{I} (kW)	γ_i^{I} (kW)	S_{min}^{I} (kW)	q_i^{II} (kW)	γ_i^{II} (kW)	S_{min}^{II} (kW)	q_i^{C} (kW)	γ_i^{C} (kW)	S_{min}^{CP} (kW)
300	0	0	**404.8**	100.0	100.0	**600.0**	100.0	100.0	**939.6**
310	0	0		−195.0	−95.0		−195.0	−95.0	
271	7.0	7		−5.0	−100.0		2.0	−93.0	
270	−231.0	−224		−105.0	−205.0		−336.0	−429.0	
249	−30.7	−254.7		−60.0	−265.0		−90.7	−519.7	
237	−69.0	−323.7		−35.0	−300.0		−104.0	−623.7	
230	−29.6	−353.3		−90.0	−390.0		−119.6	−743.3	
227	13.7	−339.6		−210.0	−600.0		−196.3	−939.6	
220	78.4	−261.2		**GAP**			78.4	−261.2	
180	−143.7	−404.8		600.0	0.0		456.3	−404.8	
160	249.9	−155.0		420.0	420.0		669.9	265.0	
146	86.8	−68.2		240.0	660.0		326.8	591.8	
138	7.2	−61.0		90.0	750.0		97.2	689.0	
135	184.4	123.4		570.0	1,320.0		754.4	1,443.4	
116	113.1	236.5	W_{min}^{I}	180.0	1,500.0	W_{min}^{II}	293.1	1,736.5	W_{min}^{CP}
110	47.3	283.8	(kW)	120.0	1,620.0	(kW)	167.3	1,903.8	(kW)
106	0	283.8		780.0	2,400.0		780.0	2,683.8	
80	0	283.8	**688.6**	−900.0	1,500.0	**2,100.0**	−900.0	**1,783.8**	**2,723.3**

Table 16. Adjusted Cascaded Heats for Example 5

	Trivedi		Ciric and Floudas	
Interval	q_k^{I} (kW)	θ_k^{I} (kW)	q_k^{II} (kW)	θ_k^{II} (kW)
1	0	353.3	100.0	686.3
2	0	353.3	−195.0	491.3
3	7.0	360.3	−5.0	486.3
4	−231.0	129.3	−105.0	381.3
5	−30.7	98.6	−60.0	321.3
6	−69.0	29.6	−35.0	286.3
7	−29.6	0	−90.0	196.3
8	13.7	13.7	−210.0	−13.7
9	78.4	—	**GAP**	—
10	−143.7	−65.3	600.0	600.0

the realization of maximum savings. Figure 17b shows how the same method can be used for the indirect transfer of heat between the plants. The vertical line in the profile of plant 2 represents the gap where no integration is possible. Still, an amount of heat can be transferred from plant 1 to plant 2 inside the pocket to allow the realization of the maximum savings for the indirect integration.

Example 6

This example consists of a crude unit processing 150,000 bbl/d (24 ML/d) and an FCC plant processing 40,000 bbl/d (64 ML/d). The crude unit is plant 1, while the FCC unit is plant 2. The data for the separate plants are shown in Table 17. The $\Delta T_{\min}$ in this case is 5.6°C (equivalent to 10°F) for both plants, and the downward shift of plant 2 during intermediate fluid integration is also 5.6°C. Pinch temperatures and minimum utility consumption are shown in Table 18 and is the result of solving problem 1 for each of the plants.

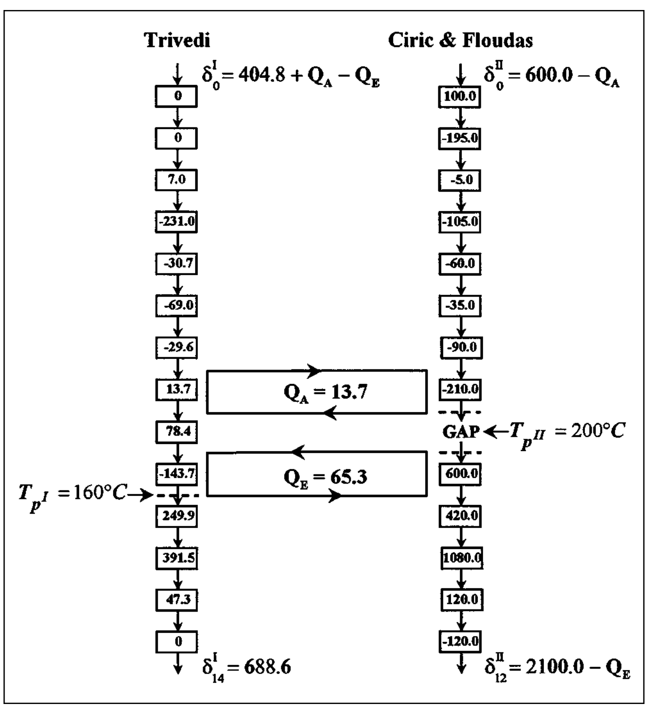

Figure 16. Assisted-circuit solution for Example 5.

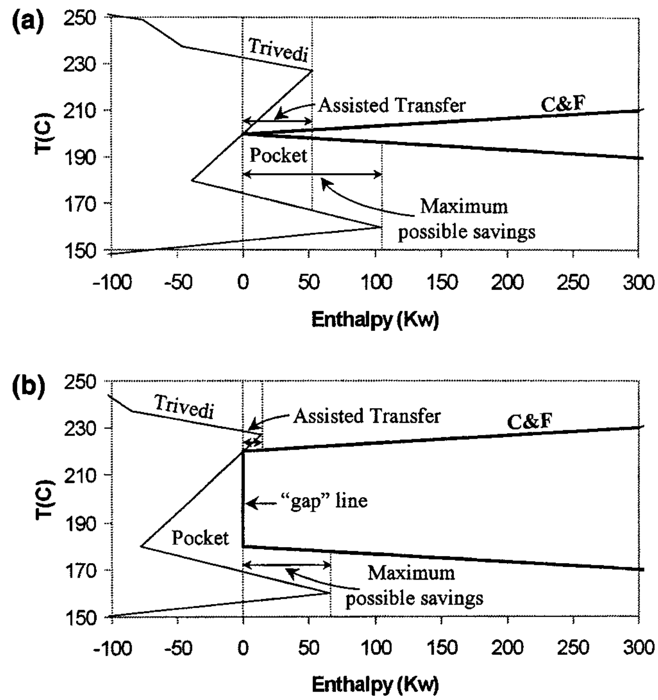

Figure 17. Countercurrent composite curve profiles for Example 5.

Direct Integration Solution. Considerable energy recovery is possible due to the large temperature difference between pinches (471.1°C to 143.5°C). The FCC unit needs a large amount of cold utility below its pinch, due to the high tem-

Table 17. Data for Example 6

Streams	F (MW/°C)	T_s (°C)	T_t (°C)	Q (MW)
Crude Unit				
C1 (Cold)	0.6230	30.0	127.3	60.64
C2 (Cold)	0.6945	127.3	239.3	77.78
C3 (Cold)	0.7855	239.3	352.9	89.24
H1 (Hot)	0.0655	127.3	37.8	5.86
H2 (Hot)	0.3053	143.5	26.7	35.67
H3 (Hot)	0.1439	261.4	37.8	32.18
H4 (Hot)	0.0334	326.7	37.8	9.64
H5 (Hot)	0.3400	347.3	268.3	26.85
H6 (Hot)	0.2744	163.3	79.6	22.98
H7 (Hot)	0.1771	194.5	142.6	9.20
H8 (Hot)	0.2617	261.4	206.3	14.42
H9 (Hot)	0.1221	336.3	239.8	11.78
F (Fuel)	124.2856	427.2	426.7	69.05
CW (Water)	0.8968	15.6	26.7	9.96
	$\Delta T_{\min} = 5.6$°C			
FCC Unit				
C4 (Cold)	0.0831	471.1	532.2	5.07
H10 (Hot)	0.0083	348.2	21.1	2.73
H11 (Hot)	0.0078	243.9	21.1	1.73
H12 (Hot)	0.0773	147.2	48.9	7.59
H13 (Hot)	0.0252	348.2	115.5	5.86
H14 (Hot)	0.0362	313.2	232.2	2.93
H15 (Hot)	0.1503	190.1	107.2	12.46
F (Fuel)	9.1262	538.3	537.8	5.07
CW (Water)	2.9990	15.6	26.7	33.32
	$\Delta T_{\min} = 5.6$°C			

Table 18. Individual Plant Pinch Analysis for Example 6

Plant	Pinch Temp. (°C)	Heating Utility (MW)	Cooling Utility (MW)
Crude unit	143.5	69.0	10.0
FCC unit	471.1	5.1	33.3

peratures of the streams emanating from the reactor. On the other hand, the crude unit needs a large amount of hot utility in order to heat up its streams during the fractionation process. Table 19 shows the results of the pinch analysis for the direct integration. The intervals below the lower pinch have been merged since heat recovery is not performed here. The resulting combined pinch is located at 163.3°C, and is the result of a compensation for the heat in the first interval of plant 1 by heat provided by plant 2. After this interval, the heat that plant 2 has available in the rest of the intervals between pinches is not sufficient to supply the demand of the corresponding intervals of plant 1. Therefore, the maximum possible heat savings are 15.1 MW. Note that 13.8 MW are transferred above the combined pinch, and 1.3 MW below the combined pinch.

Indirect Integration. Table 20 shows the results of the pinch analysis for the indirect integration. After the hot temperature scale in plant 2 is shifted down 18°C, the interval between pinches is from 465.6°C to 143.5°C. The maximum possible savings are now 13.9 MW. This amount is 1.2 MW smaller than the maximum possible heat found for the direct integration.

Table 19. Pinch Analysis for Direct Integration in Example 6

	Crude Unit			FCC Unit			Combined Plant		
T (°C)	q_i^I (MW)	γ_i^I (MW)	S_{min}^I (MW)	q_i^{II} (MW)	γ_i^{II} (MW)	S_{min}^{II} (MW)	q_i^C (MW)	γ_i^C (MW)	S_{min}^{CP} (MW)
532.2	0	0	**69.1**	−5.1	−5.1	**5.1**	−5.1	−5.1	**59.0**
471.1	−8.1	−8.1		0.0	−5.1		−8.1	−13.2	
348.2	−0.7	−8.8		0.0	−5.0		−0.7	−13.9	
347.3	−4.9	−13.7		0.4	−4.7		−4.5	−18.4	
336.3	−3.1	−16.8		0.3	−4.4		−2.8	−21.2	
326.7	−3.9	−20.7		0.5	−3.9		−3.5	−24.6	
313.2	−13.0	−33.7		3.1	−0.8		−9.9	−34.5	
268.3	−4.3	−38.1		0.5	−0.3		−3.9	−38.4	
261.4	−3.7	−41.8		1.2	0.9		−2.6	−40.9	
244.9	−0.1	−41.9		0.1	0.9		−0.1	−41.0	
243.9	−0.5	−42.5		0.3	1.2		−0.2	−41.2	
239.8	−1.9	−44.4		0.6	1.8		−1.3	−42.6	
232.2	−6.6	−51.0		1.1	2.9		−5.5	−48.1	
206.3	−6.1	−57.1		0.5	3.4		−5.6	−53.8	
194.5	−1.5	−58.6		0.2	3.6		−1.3	−55.1	
190.1	−9.1	−67.8		5.1	8.7		−4.0	−59.0	
163.3	−1.1	−68.8		3.1	11.8		2.0	−57.0	
147.2	−0.2	−69.1	W_{min}^I (kW)	1.0	12.8	W_{min}^{II} (kW)	0.7	−56.3	W_{min}^{CP} (kW)
143.5	0.2	−68.8		0.3	13.0		0.5	−55.8	
26.7	9.8	−59.1	**10.0**	15.2	28.2	**33.3**	24.9	−30.9	**28.2**

Table 20. Pinch Analysis for Indirect Integration in Example 6

	Crude Unit			FCC Unit			Combined Plant		
T (°C)	q_i^I (MW)	γ_i^I (MW)	S_{min}^I (MW)	q_i^{II} (MW)	γ_i^{II} (MW)	S_{min}^{II} (MW)	q_i^C (MW)	γ_i^C (MW)	S_{min}^{CP} (MW)
526.7	0	0	**69.1**	−5.1	−5.1	**5.1**	−5.1	−5.1	**60.1**
465.6	−8.8	−8.8		0	−5.1		−8.8	−13.9	
347.3	−2.1	−10.9		0	−5.1		−2.1	−15.9	
342.7	−2.8	−13.7		0.2	−4.9		−2.6	−18.6	
336.3	−3.1	−16.8		0.3	−4.5		−2.8	−21.3	
326.7	−5.5	−22.3		0.6	−3.9		−4.9	−26.2	
307.7	−11.4	−33.7		2.7	−1.2		−8.7	−34.9	
268.3	−4.3	−38.1		0.5	−0.7		−3.9	−38.8	
261.4	−3.7	−41.8		1.2	0.5		−2.6	−41.3	
244.9	−0.7	−42.5		0.4	0.8		−0.3	−41.6	
239.8	−0.4	−42.8		0.1	0.9		−0.3	−41.9	
238.3	−3.0	−45.8		0.9	1.8		−2.1	−44.0	
226.7	−5.2	−51.0		0.8	2.7		−4.4	−48.3	
206.3	−6.1	−57.1		0.5	3.2		−5.6	−54.0	
194.5	−3.4	−60.5		0.4	3.6		−3.0	−56.9	
184.6	−7.2	−67.8		4.1	7.6		−3.2	−60.1	
163.3	−1.3	−69.1	W_{min}^I (kW)	3.8	11.4	W_{min}^{II} (kW)	2.5	−57.6	W_{min}^{CP} (kW)
143.5	0.2	−68.8		0.2	11.6		0.4	−57.2	
142.6	9.8	−59.1	**10.0**	16.6	28.2	**33.3**	26.4	−30.8	**29.3**

Figure 18 shows the result of the higher-circuit solution. Without loss of generality, some of the intervals have been lumped to clarify the illustration in the upper part of the zone between pinches and below the pinch of plant 1. This

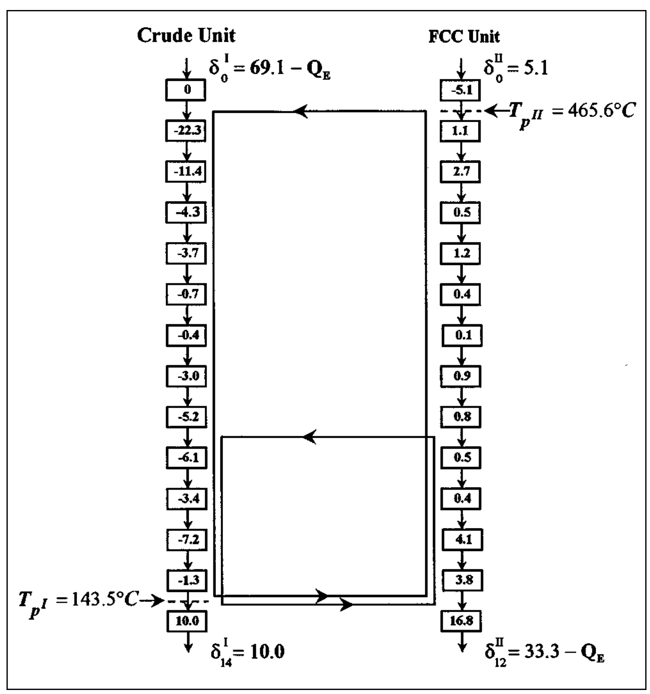

Figure 18. Higher and lower candidate single-circuit solutions for Example 6.

candidate solution covers all the intervals and in plant 2 transfers all the heat in each interval except in the last one, where a lower amount of heat is transferred. In this last interval the value of the maximum possible heat Q_E is reached. The lower-circuit solution is also shown in Figure 18. Only four intervals are required to transfer the maximum possible heat. In this case, the test with Eqs. 38 and 39 fails for both candidate solutions. Then, a single circuit will transfer a smaller amount of heat than the maximum Q_E. This is determined next.

Maximum Amount Transferred by a Single Circuit

In the case where a single circuit cannot realize the entire target savings, one can still consider establishing a single circuit and realize only a portion of these total savings. Consider the unassisted case first. The maximum amount of heat transferred by a single circuit, for which its location (starting and ending intervals) is known, can be obtained by solving the following problem:

$$\mathrm{Min}\left(\delta_0^{\mathrm{I}}+\delta_m^{\mathrm{II}}\right)$$

s.t.

$$\delta_0^{\mathrm{I}}=\hat{\delta}_0^{\mathrm{I}}-Q_E$$

$$\delta_0^{\mathrm{II}}=\hat{\delta}_0^{\mathrm{II}}$$

$$\delta_i^j=\delta_{i-1}^j+q_i^j \qquad \forall i=1,\ldots,(k^E-1);\; j=\mathrm{I},\mathrm{II}$$

$$\left.\begin{aligned}\delta_i^{\mathrm{I}}&=\delta_{i-1}^{\mathrm{I}}+q_i^{\mathrm{I}}+q_i^{EH}\\ \delta_i^{\mathrm{II}}&=\delta_{i-1}^{\mathrm{II}}+q_i^{\mathrm{II}}-q_i^{EC}\end{aligned}\right\} \qquad \forall i=k^E,\ldots,\left(k^E+m_E\right)$$

$$\delta_i^j=\delta_{i-1}^j+q_i^j \qquad \forall i=\left(k^E+m_E+1\right),\ldots,m;\quad j=\mathrm{I},\mathrm{II}$$

$$\delta_m^{\mathrm{I}}=\hat{\delta}_m^{\mathrm{I}}$$

$$\delta_m^{\mathrm{II}}=\hat{\delta}_m^{\mathrm{II}}-Q_E$$

$$\delta_{p^{\mathrm{I}}}^{\mathrm{I}}=0,\quad \delta_{p^{\mathrm{II}}}^{\mathrm{II}}=0 \tag{40}$$

$$F_E\sum_{i=k^E}^{k}\Delta T_i\geq\sum_{i=k^E}^{k}q_i^{EH} \qquad \forall k=k^E,\ldots,\left(k^E+m_E-1\right)$$

$$F_E\sum_{i=k^E}^{k^E+m_E}\Delta T_i=\sum_{i=k^E}^{k^E+m_E}q_i^{EH}$$

$$F_E\sum_{i=k}^{k^E+m_E}\Delta T_i\geq\sum_{i=k}^{k^E+m_E}q_i^{EC} \qquad \forall k=(k^E+1),\ldots,\left(k^E+m_E\right)$$

$$F_E\sum_{i=k^E}^{k^E+m_E}\Delta T_i=\sum_{i=k^E}^{k^E+m_E}q_i^{EC}$$

$$\delta_i^{\mathrm{I}},\ \delta_i^{\mathrm{II}},\ q_i^{EH},\ q_i^{EC}\geq 0,$$

where $\Delta T_i=T_{i-1}-T_i$.

The equalities that correspond to the heat balances in each interval have been split into two sets of equalities. The first one considers only those intervals in which all the heat cascades down. The second set consists of the intervals in which the transfer is taking place. This problem is linear and offers no major difficulties.

For assisted cases, the set of equations for the balances in each interval in problem 40 are replaced by the following new sets:

$$\delta_i^j=\delta_{i-1}^j+q_i^j \qquad \forall i=1,\ldots,(k^A-1);\; j=\mathrm{I},\mathrm{II}$$

$$\left.\begin{aligned}\delta_i^{\mathrm{I}}&=\delta_{i-1}^{\mathrm{I}}+q_i^{\mathrm{I}}+q_i^{AC}\\ \delta_i^{\mathrm{II}}&=\delta_{i-1}^{\mathrm{II}}+q_i^{\mathrm{II}}-q_i^{AH}\end{aligned}\right\} \qquad \forall i=k^A,\ldots,\left(k^A+m_A\right) \tag{41}$$

$$\delta_i^j=\delta_{i-1}^j+q_i^j \qquad \forall i=\left(k^A+m_A+1\right),\ldots,(k^E-1);\quad j=\mathrm{I},\mathrm{II}$$

$$\delta_i^j=\delta_{i-1}^j+q_i^j \qquad \forall i=\left(k^E+m_E+1\right),\ldots,(k^B-1);\quad j=\mathrm{I},\mathrm{II}$$

$$\left.\begin{aligned}\delta_i^{\mathrm{I}}&=\delta_{i-1}^{\mathrm{I}}+q_i^{\mathrm{I}}+q_i^{BC}\\ q_i^{\mathrm{II}}&=\delta_{i-1}^{\mathrm{II}}+q_i^{\mathrm{II}}-q_i^{BH}\end{aligned}\right\} \qquad \forall i=k^B,\ldots,\left(k^B+m_B\right) \tag{42}$$

$$\delta_i^j=\delta_{i-1}^j+q_i^j \qquad \forall i=\left(k^B+m_B+1\right),\ldots,m;\quad j=\mathrm{I},\mathrm{II}.$$

In addition to these equations, the following two new sets of constraints have to be added in order to account for the feasibility of a single circuit in each of the aforementioned regions:

$$F_A\sum_{i=k^A}^{k}\Delta T_i\geq\sum_{i=k^A}^{k}q_i^{AC} \qquad \forall k=k^A,\ldots,\left(k^A+m_A-1\right)$$

$$F_A\sum_{i=k^A}^{k^A+m_A}\Delta T_i=\sum_{i=k^A}^{k^A+m_A}q_i^{AC}$$

$$F_A\sum_{i=k}^{k^A+m_A}\Delta T_i\geq\sum_{i=k}^{k^A+m_A}q_i^{AH} \qquad \forall k=(k^A+1),\ldots,\left(k^A+m_A\right)$$

$$F_A\sum_{i=k^A}^{k^A+m_A}\Delta T_i=\sum_{i=k^A}^{k^A+m_A}q_i^{AH} \tag{43}$$

$$F_B\sum_{i=k^B}^{k}\Delta T_i\geq\sum_{i=k^B}^{k}q_i^{BC} \qquad \forall k=k^B,\ldots,\left(k^B+m_B-1\right)$$

$$F_B\sum_{i=k^B}^{k^B+m_B}\Delta T_i=\sum_{i=k^B}^{k^B+m_B}q_i^{Bc}$$

$$F_B\sum_{i=k}^{k^B+m_B}\Delta T_i\geq\sum_{i=k}^{k^B+m_B}q_i^{BH} \qquad \forall k=(k^B+1),\ldots,\left(k^B+m_B\right)$$

$$F_B\sum_{i=k^B}^{k^B+m_B}\Delta T_i=\sum_{i=k^B}^{k^B+m_B}q_i^{BH}. \tag{44}$$

Optimum Location of a Single Circuit

Problem 40 is expressed for known fixed starting and ending intervals. An MILP formulation to determine the optimum points of insertion of such a circuit in any of the regions is presented next. The case of a circuit between pinches is presented.

Consider the following general binary variables:

$$Y_i^{FH} = \begin{cases} 1 & \text{Interval } (i+1) \text{ is the starting interval} \\ 0 & \text{Otherwise} \end{cases}$$

$$Y_i^{FC} = \begin{cases} 1 & \text{Interval } i \text{ is the ending interval} \\ 0 & \text{Otherwise.} \end{cases}$$

To guarantee that only an interval is a starting/ending one, the following inequalities are introduced:

$$\sum_{i=p^{\mathrm{II}}}^{p^{\mathrm{I}}-1} Y_i^{FH} = 1 \tag{45}$$

$$\sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} Y_i^{FC} = 1. \tag{46}$$

In addition, the following variables are defined:

$$Z_i^F = \begin{cases} 1 & \text{Interval } i \text{ is in the circuit} \\ 0 & \text{Otherwise.} \end{cases}$$

These variables are related to Y_i^{FH} and Y_i^{FC} by the following equalities:

$$Z_{p^{\mathrm{II}}+1}^F = Y_{p^{\mathrm{II}}}^{FH} \tag{47}$$

$$Z_i^F = Z_{i-1}^F + Y_{i-1}^{FH} - Y_{i-1}^{FC} \quad \forall i = (p^{\mathrm{II}}+2), \ldots, p^{\mathrm{I}}. \tag{48}$$

These equalities are needed to restrict the values of the heat transferred to and from the intermediate fluid to zero for intervals that are not in the circuit. For example, consider a circuit starting in the first interval below the pinch temperature of plant 2. Then

$$Y_{p^{\mathrm{II}}}^{FC} = 1 \quad \text{and} \quad Z_{p^{\mathrm{II}}+1}^F = Y_{p^{\mathrm{II}}}^{FH} = 1;$$

otherwise, $Y_{p^{\mathrm{II}}}^{FH} = 0$ and the first interval of transfer will be located at a low level. That is, $Z_{p^{\mathrm{II}}+1}^F = 0$. Let's now consider that the third interval is the starting one. Then

$$Y_{p^{\mathrm{II}}+2}^{FC} = 1 \quad \text{and}$$

$$Z_{p^{\mathrm{II}}+3}^F = Z_{p^{\mathrm{II}}+2}^F + Y_{p^{\mathrm{II}}+2}^{FH} - Y_{p^{\mathrm{II}}+2}^{FC} = 0+1+0=1.$$

In any case, $Y_{p^{\mathrm{II}}}^{FC}$ must be zero in the interval in which $Y_{p^{\mathrm{II}}}^{FH}$ is one in order for the circuit to span at least an interval. Finally, consider that the circuit ends in the fifth interval. Then

$$Y_{p^{\mathrm{II}}+5}^{FC} = 1 \quad \text{and}$$

$$Z_{p^{\mathrm{II}}+6}^F = Z_{p^{\mathrm{II}}+5}^F + Y_{p^{\mathrm{II}}+5}^{FH} - Y_{p^{\mathrm{II}}+5}^{FC} = 1+0-1=0.$$

Therefore, there will not be a transfer in the sixth interval.

An optimization problem based on the preceding binary variables to solve the unassisted case is then proposed.

$$\begin{aligned}
&\mathrm{Min}\left(\delta_0^{\mathrm{I}} + \delta_m^{\mathrm{II}}\right) \\
&\text{s.t.} \\
&\delta_0^{\mathrm{I}} = \hat{\delta}_0^{\mathrm{I}} - Q_E \\
&\delta_0^{\mathrm{II}} = \hat{\delta}_0^{\mathrm{II}} \\
&\delta_i^j = \delta_{i-1}^j + q_i^j \qquad \forall i = 1, \ldots, p^{\mathrm{II}};\ j = \mathrm{I}, \mathrm{II} \\
&\left.\begin{aligned} \delta_i^{\mathrm{I}} &= \delta_{i-1}^{\mathrm{I}} + q_i^{\mathrm{I}} + q_i^{EH} \\ \delta_i^{\mathrm{II}} &= \delta_{i-1}^{\mathrm{II}} + q_i^{\mathrm{II}} - q_i^{EC} \end{aligned}\right\} \qquad \forall i = (p^{\mathrm{II}}+1), \ldots, p^{\mathrm{I}} \\
&\delta_i^j = \delta_{i-1}^j + q_i^j \qquad \forall i = (p^{\mathrm{I}}+1), \ldots, m;\ j = \mathrm{I}, \mathrm{II} \\
&\delta_m^{\mathrm{I}} = \hat{\delta}_m^{\mathrm{I}} \\
&\delta_m^{\mathrm{II}} = \hat{\delta}_m^{\mathrm{II}} - Q_E \\
&\delta_{p^{\mathrm{I}}}^{\mathrm{I}} = 0 \\
&\delta_{p^{\mathrm{II}}}^{\mathrm{II}} = 0 \\
&F_E \sum_{i=p^{\mathrm{II}}+1}^{k} Z_i^E \Delta T_i \geq \sum_{i=p^{\mathrm{II}}+1}^{k} q_i^{EH} \qquad \forall k = (p^{\mathrm{II}}+1), \ldots, (p^{\mathrm{I}}-1) \\
&F_E \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} Z_i^E \Delta T_i = \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} q_i^{EH} \\
&F_E \sum_{i=k}^{p^{\mathrm{I}}} Z_i^E \Delta T_i \geq \sum_{i=k}^{p^{\mathrm{I}}} q_i^{EC} \qquad \forall k = (p^{\mathrm{II}}+2), \ldots, p^{\mathrm{I}} \\
&F_E \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} Z_i^E \Delta T_i = \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} q_i^{EC} \\
&q_i^{EH} - U Z_i^E \leq 0 \qquad \forall i = (p^{\mathrm{II}}+1), \ldots, p^{\mathrm{I}} \\
&q_i^{EC} - U Z_i^E \leq 0 \qquad \forall i = (p^{\mathrm{II}}+1), \ldots, p^{\mathrm{I}} \\
&Z_{p^{\mathrm{II}}+1}^E = Y_{p^{\mathrm{II}}}^{EH} \\
&Z_i^E = Z_{i-1}^E + Y_{i-1}^{EH} - Y_{i-1}^{EC} \qquad \forall i = (p^{\mathrm{II}}+2), \ldots, p^{\mathrm{I}} \\
&\sum_{i=p^{\mathrm{II}}}^{p^{\mathrm{I}}-1} Y_i^{EH} = 1 \\
&\sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} Y_i^{EC} = 1 \\
&\delta_i^{\mathrm{I}},\ \delta_i^{\mathrm{II}},\ q_i^{EH},\ q_i^{EC},\ Z_i^E \geq 0 \\
&Y_i^{EH},\ Y_i^{EC} \in \{0,1\}.
\end{aligned} \tag{49}$$

In these equations, U is an upper bound of the total heat that can be transferred. This is a mixed-integer nonlinear problem having a single nonlinearity that consists of the product of a continuous variable times a binary variable. The following constraints are introduced to eliminate this nonlinearity:

$$\left.\begin{array}{l} B_i = F^E Z_i^E \\ F^E \geq 0 \\ Z_i^E = (0,1) \end{array}\right\} \Leftrightarrow \left\{\begin{array}{c} B_i - Z_i^E \Omega \leq 0 \\ B_i \geq 0 \\ \left(F^E - B_i\right) - \left(1 - Z_i^E\right)\Omega \leq 0 \\ \left(F^E - B_i\right) \geq 0 \\ Z_i^E = (0,1), \end{array}\right. \quad (50)$$

where Ω is a sufficiently large number.

Assisted cases can also be solved by introducing similar constraints in the appropriate temperature intervals.

Example 6 (continued)

The linear formulation of problem 49 by introducing Eq. 50 is implemented in GAMS (Brooke et al., 1996). The MILP model obtained is solved with the CPLEX solver. The two optimum solutions found are shown in Table 21 and Figure 19. Any of the circuits is capable of transferring 12.6 MW, which represents 91% of the total possible savings predicted by problem 1. Since a single circuit is not capable of transferring the maximum possible heat, a new formulation is presented that achieves this target with the minimum number of circuits.

Optimum Location of Many Circuits

When a single circuit is not capable of realizing the maximum target savings, a step-by-step increase of the number of circuits seems a logical procedure to reach the minimum required. Then at each step the optimal location of an increasing number of circuits is to be found maximizing the overall heat transfer. The following modification of Eq. 49 is proposed in order to find the locaton of a number n of circuits:

$$\mathrm{Min}\left(\delta_0^{\mathrm{I}} + \delta_m^{\mathrm{II}}\right)$$

s.t.

$$\delta_0^{\mathrm{I}} + \hat{\delta}_0^{\mathrm{I}} - Q_E^{(l)}$$

$$\delta_0^{\mathrm{II}} = \hat{\delta}_0^{\mathrm{II}}$$

$$\delta_i^j = \delta_{i-1}^j + q_i^j \qquad \forall i = 1, \ldots, p^{\mathrm{II}};\ j = \mathrm{I}, \mathrm{II}$$

$$\left.\begin{array}{l} \delta_i^{\mathrm{I}} = \delta_{i-1}^{\mathrm{I}} + q_i^{\mathrm{I}} + \sum_{l=1}^{n} q_{i,l}^{EH} \\ \delta_i^{\mathrm{II}} = \delta_{i-1}^{\mathrm{II}} + q_i^{\mathrm{II}} - \sum_{l=1}^{n} q_{i,l}^{EC} \end{array}\right\} \quad \forall i = \left(p^{\mathrm{II}} + 1\right), \ldots, p^{\mathrm{I}}$$

$$\delta_i^j = \delta_{i-1}^j + q_i^j \qquad \forall i = \left(p^{\mathrm{I}} + 1\right), \ldots, m;\ j = \mathrm{I}, \mathrm{II}$$

$$\delta_m^{\mathrm{I}} = \hat{\delta}_m^{\mathrm{I}}$$

$$\delta_m^{\mathrm{II}} = \hat{\delta}_m^{\mathrm{II}} - Q_E^{(l)}$$

$$\delta_{p^{\mathrm{I}}}^{\mathrm{I}} = 0$$

$$\delta_{p^{\mathrm{II}}}^{\mathrm{II}} = 0 \qquad (51)$$

$$\left.\begin{array}{l} F_E^{(l)} \sum_{i=p^{\mathrm{II}}+1}^{k} Z_{i,l}^E \Delta T_i \geq \sum_{i=p^{\mathrm{II}}+1}^{k} q_{i,l}^{EH} \\ \qquad \forall k = \left(p^{\mathrm{II}}+1\right), \ldots, \left(p^{\mathrm{I}}-1\right) \\ F_E^{(l)} \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} Z_{i,l}^E \Delta T_i = \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} q_{i,l}^{EH} \\ F_E^{(l)} \sum_{i=k}^{p^{\mathrm{I}}} Z_{i,l}^E \Delta T_i \geq \sum_{i=k}^{p^{\mathrm{I}}} q_{i,l}^{EC} \\ \qquad \forall k = \left(p^{\mathrm{II}}+2\right), \ldots, p^{\mathrm{I}} \\ F_E^{(l)} \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} Z_{i,l}^E \Delta T_i = \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} q_{i,l}^{EC} \\ q_{i,l}^{EH} - U Z_{i,l}^E \leq 0 \quad \forall i = \left(p^{\mathrm{II}}+1\right), \ldots, p^{\mathrm{I}} \\ q_{i,l}^{EC} - U Z_{i,l}^E \leq 0 \quad \forall i = \left(p^{\mathrm{II}}+1\right), \ldots, p^{\mathrm{I}} \\ Z_{p^{\mathrm{II}}+1,l}^E = Y_{p^{\mathrm{II}},l}^{EH} \\ Z_{i,l}^E = Z_{i-1,l}^E + Y_{i-1,l}^{EH} - Y_{i-1,l}^{EC} \\ \qquad \forall i = \left(p^{\mathrm{II}}+2\right), \ldots, p^{\mathrm{I}} \\ \sum_{i=p^{\mathrm{II}}}^{p^{\mathrm{I}}-1} Y_{i,l}^{EH} = 1 \\ \sum_{i=p^{\mathrm{II}}+1}^{p^{\mathrm{I}}} Y_{i,l}^{FC} = 1 \end{array}\right\} \forall l = 1, \ldots, n$$

$$\delta_i^{\mathrm{I}},\ \delta_i^{\mathrm{II}},\ q_{i,l}^{EH},\ q_{i,l}^{EC},\ Z_{i,l}^E \geq 0$$

$$Y_{i,l}^{EH},\ Y_{i,l}^{EC} \in \{0,1\}.$$

The strategy to find the optimum number of circuits consists of a trial procedure. At each step, the value obtained by solving Eq. 51 is compared with the maximum heat possible to be transferred. If the difference is not zero, then the value of l is increased to approach the target. The minimum number of circuits resulting from this procedure will be less than or equal to the number of intervals between pinches.

Example 6 (continued)

If the formulation just presented is applied to Example 6, a minimum of two circuits is obtained. This set of two circuits will transfer all the heat predicted by problem 1. The location of both circuits for one of the possible solutions (first alternative) to problem 51 is shown in Figure 20. As illustrated, this solution is the combination of one of the possible solutions obtained for a single circuit (covering four intervals)

Table 21. Single Circuit Solutions to Example 6

Solution	No. of Intervals	T_{up} (°C)	T_{down} (°C)	F (MW/°C)
Optimum #1	4	226.7	163.3	0.199
Optimum #2	3	206.3	163.3	0.293

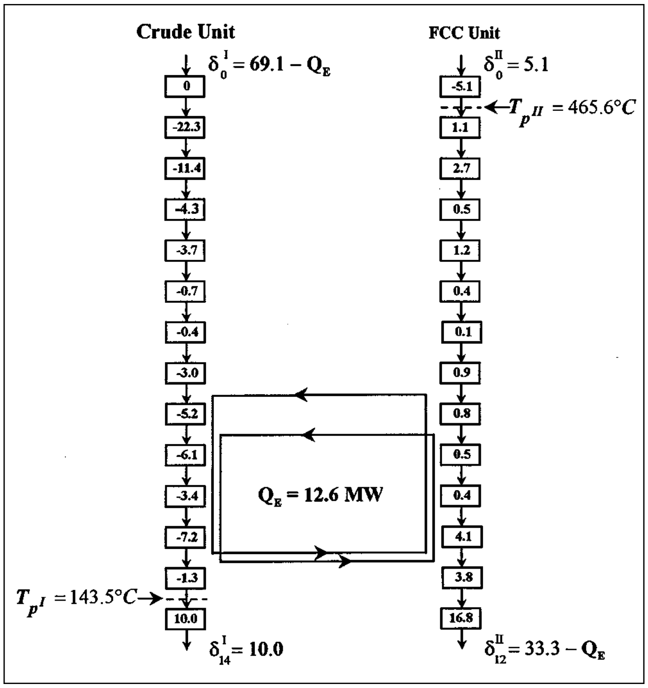

Figure 19. Two alternative single-circuit solutions for Example 6.

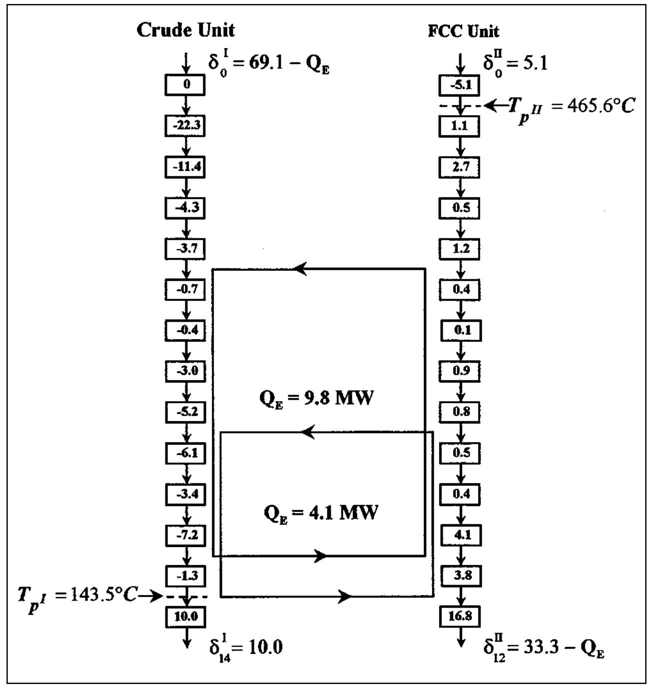

Figure 21. Two-circuits solutions for Example 6 (second alternative).

and a circuit covering the last interval. Another possible solution (second alternative) is shown in Figure 21. In this alternative, the two circuits overlap. Due to degeneracy, there are a large number of possible solutions. The two alternatives considered here were obtained using GAMS with CPLEX.

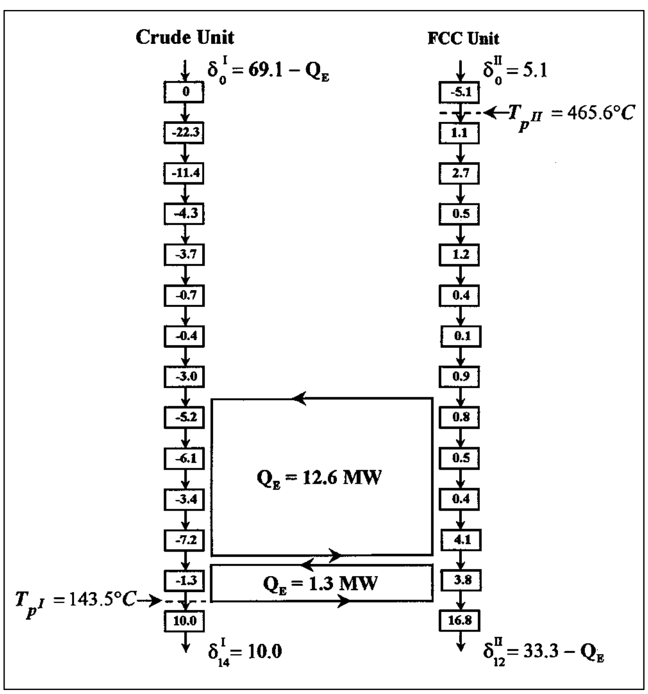

Figure 20. Two-circuits solutions for Example 6 (first alternative).

Indirect Integration Using Steam

The use of steam for indirect integration imposes extra restrictions on the maximum amount of heat that can be transferred. Consider that steam at fixed pressure levels is generated by the plant having an excess of heat (higher pinch temperature). For simplicity, assume first that a single steam-temperature level is specified between pinches, and that is the only indirect fluid used (Figure 22). By computing the cooling utility required for the source plant from its pinch down to the level of steam generation, the heat load of this steam can be established. This amount is the maximum heat this steam will be able to transfer to the sink plant. Consider now the sink plant and the zone between pinches. This plant will be able to use the steam coming from the source plant to reduce its heating utility demands only if this steam temperature is above its pinch temperature. In addition, the maximum load that the sink plant can accept is the deficit it presents between the steam level and the pinch. Thus, the use of latent heat of a single steam stream may reduce the opportunities of integration.

An alternative is the use of the utility system to balance the steam supply and demand of source and sink plants, respectively (Hui and Ahmad, 1994). In any case, the difficulties arise when more than one steam level is considered. Hui and Ahmad (1994) consider the utility as a "market," selling and buying utilities at fixed prices from the processes. Treating every single plant individually, they applied a procedure for multiple-utilities optimization (Parker, 1989).

Generalization for More than Two Plants

The concepts explored up to this point can be extended to the case in which a number *n* of plants is considered for

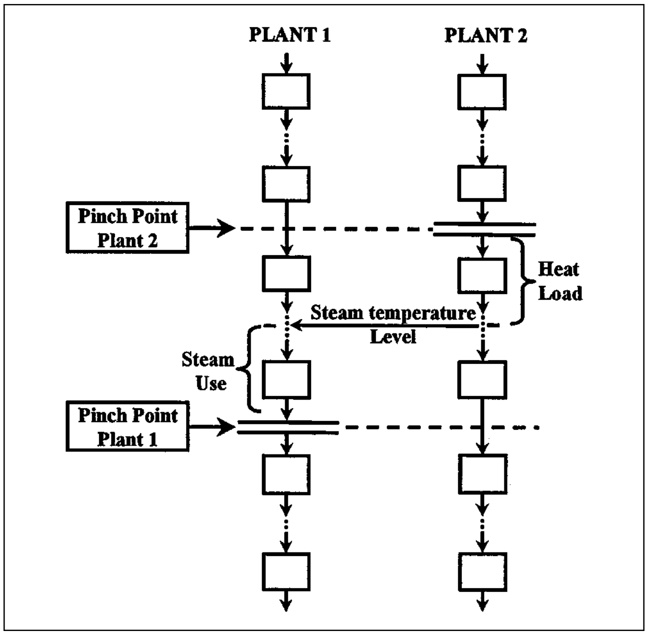

Figure 22. Indirect integration using a single-steam temperature level.

integration. The increase in complexity is evident, since in principle, all possible combinations of two plants have to be evaluated. As a starting point, consider the example of integration among three plants. First, the plants are ordered by increasing pinch temperatures, and the highest and lowest pinches are identified. For the unassisted cases, the zone delimited by these pinches is where all the integration can take place. The three possible ways of heat transfer are shown in Figure 23. In turn, assisted cases will require transfer heat in the highest or lowest zones. To predict maximum possible savings, problem 1 can be reformulated, accounting for each one of the combinations of two plants and the respective directions in which the heat will be transferred. The results of this problem are useful targets for models that will determine

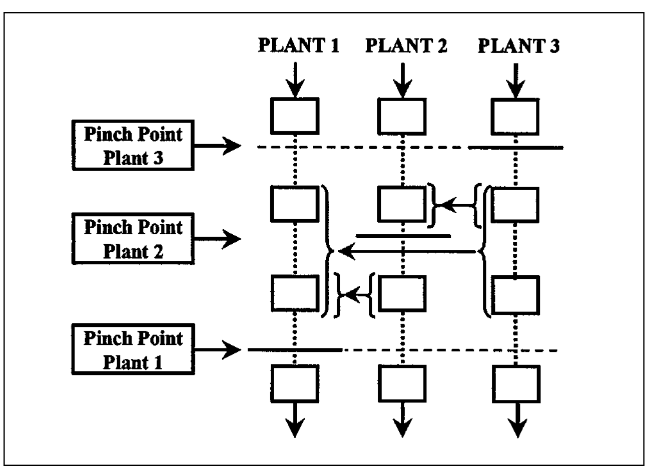

Figure 23. Integration among three plants.

the heat-exchanger network needed to accomplish the predicted savings.

In the case of indirect integration, new shifts of scales are required. As a first-approach solution, three circuits can be established, accounting for the three possible combinations of two plants in the unassisted cases. A downward shift is performed in the scale of the second and third plants to feasible transfer heat from the first plant. In turn, a second downward shift is needed in the third plant in order to transfer heat from the second plant. Other alternatives may consist of the use of a single circuit that splits in plant 2 and 3, picking up the required heat in each of these plants and performing a similar split when the heat is to be released. Therefore, the concepts and tools developed for two plants are a stepping stone for the generalized integration of a set of n plants, an issue that will be further investigated in follow-up articles.

Conclusion

There is a large incentive to perform heat integration across plants. Models that account for maximum energy savings by direct and indirect heat integration, including in this last case the location of the fluid circuits, were presented. Consequently, a strategy to capture these savings was developed. While all these studies determine the target savings, there is still a need to determine a heat-exchanger network that can accomplish minimum energy consumption while the plants are integrated, as well as when they are functioning separately. This must take place at a minimum investment cost. Among many other options, dual-use heat-exchanger networks featuring the minimum number of units accomplish this goal. The design of such networks will be attempted in a follow-up article.

Notation

F = product of heat capacity and flow rate
k = auxiliary temperature intervals
k^A = first transfer interval in the zone above both pinches
k^B = first transfer interval in the zone below both pinches
k^E = first transfer interval in the zone between pinches
p^{I} = last interval above the pinch of plant 1
p^{II} = last interval above the pinch of plant 2
Q_F = total heat transferred in the generalized zone
q = heat surplus or heat demand
q^{I} = heat surplus or heat demand in plant 1
q^{II} = heat surplus or heat demand in plant 2
q^C = heat demand in the heat-sink plant
q^{CP} = heat surplus or heat demand in the combined plant
q^H = heat surplus in the heat source plant
$S_{\min}$ = minimum heating utility
T_0 = initial temperature of the intermediate fluid
T_s = supply temperature
T_t = target temperature
T_{up} = upper temperature of a fluid circuit
T_{down} = lower temperature of a fluid circuit
Y^{FH} = binary variable starting interval of a fluid circuit
Y^{FC} = binary variable starting interval of a fluid circuit
Z = binary variable denoting an interval that belongs to a fluid circuit
$\hat{\delta}$ = original minimum cascaded heat
$\hat{\delta}^C$ = original minimum cascaded heat in the heat-sink plant
$\hat{\delta}^H$ = original minimum cascaded heat in the heat-source plant
γ = variable used in the cascade of heat
θ = cumulative heat demands
θ^{I} = adjust cascaded heat in plant 1

θ^{II} = adjust cascaded heat in plant 2
λ^{H} = heat demand in the heat source plant

Superscripts

AC = cold fluid stream in the zone above both pinches
AH = hot fluid stream in the zone above both pinches
BC = cold fluid stream in the zone below both pinches
BH = hot fluid stream in the zone below both pinches
EC = cold fluid stream in the zone of effective transfer of heat
EH = hot fluid stream in the zone of effective transfer of heat
FC = cold fluid stream in the general case
FH = hot fluid stream in the general case
j = chemical plant

Subscripts

j = chemical plant
r = hot stream
s = cold stream

Literature Cited

Ahmad, S., and C. W. Hui, ''Heat Recovery Between Areas of Integrity,'' *Comput. Chem. Eng.*, **12**, 809 (1991).

Brooke, A., D. Kendrick, and A. Meeraus, *GAMS. A User's Guide*, GAMS Development Corporation, Washington, DC (1996).

Cerda, J., A. W. Westerberg, D. Mason, and B. Linnhoff, ''Minimum Utility Usage in Heat Exchanger Network Synthesis,'' *Chem. Eng. Sci.*, **38**, 373 (1983).

Ciric, A. R., and C. A. Floudas, ''Heat Exchanger Network Synthesis Without Decomposition,'' *Comput. Chem. Eng.*, **15**, 385 (1991).

Dhole, V. R., and B. Linnhoff, ''Total Site Targets for Fuel, Co-Generation, Emissions, and Cooling,'' *Comput. Chem. Eng.*, **17**, S101 (1992).

Hui, C. W., and S. Ahmad, ''Total Site Heat Integration Using the Utility System,'' *Comput. Chem. Eng.*, **18**, 729 (1994).

Linnhoff, B., and E. Hindmarsh, ''The Pinch Design Method for Heat Exchanger Networks,'' *Chem. Eng. Sci.*, **38**, 745 (1983).

Morton, R. J., and B. Linnhoff, ''Individual Process Improvements in the Context of Site-wide Interactions,'' *Inst. Chem. Eng.*, Annual Research Meeting, Bath, UK (1984).

Papoulias, S., and I. E. Grossmann, ''A Structural Optimization Approach in Process Synthesis—II: Heat Recovery Networks,'' *Comput. Chem. Eng.*, **7**, 707 (1983).

Parker, S. J., ''Supertargeting for Multiple Utilities,'' PhD Thesis, Univ. of Manchester Institute of Science and Technology, Manchester, UK (1989).

Trivedi, K. K., ''The Pinch Design Method for the Synthesis of Heat Exchanger Networks: The Constrained Case,'' AIChE Meeting, Washington, DC (1988).

Manuscript received Aug. 17, 1998, and revision received May 20, 1999.